PROVA IN ITINERE: 18 APRILE CIRCA 3/3/09

Esame scritto con esercizi e teoria.

Il controllo attivo ha come obiettivo l'imposizione di una modalità di funzionamento desiderato del processo.

La modalità di funzionamento viene identificata da una variabile.

VARIABILI CONTROLLATE → regolate a un certo valore

VARIABILI MANIPOLABILI → quelle disponibili all'elaborazione

VARIABILI NON MANIPOLABILI → disturbi

VARIABILI OSSERVATE → misurate in tempo reale

Nei sistemi orientati, l'uscita si ottiene moltiplicando l'ingresso per il guadagno.

SISTEMI SCALARI (SISO) → un ingresso, un'uscita

SISTEMI MULTIVARIABILI (MIMO) → più ingressi, più uscite

SISTEMA STATICO → l'uscita all'istante t dipende solo dal t-esimo ingresso $y(t) = f(u(t))$

SISTEMA DINAMICO → l'uscita all'istante t dipende dall'intera storia passata del segnale d'ingresso.

sistemi con memoria

Vengono descritti da equazioni differenziali o da integrali di convoluzione (funzionali).

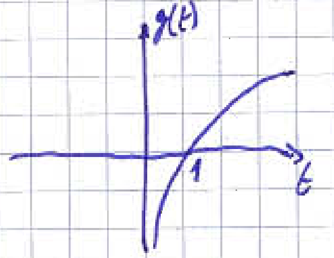
INSIEME DI BEHAVIOURS (comportamenti) → insieme di tutte le possibili coppie ingresso-uscita associate al sistema.

Se è uno spazio vettoriale, il sistema è lineare.

Se l'azione di comando dipende anche dalla variabile controllata, siamo in presenza di retroazione.

La retroazione permette di contrastare disturbi o perturbazioni più efficacemente.

MODELLISTICA → scienza per mettere in atto i sistemi dinamici 5/3/09



$$g(t) = \begin{cases} 0 & t \leq 0 \\ \ln t & t > 0 \end{cases} \qquad f(t) = \int_0^t g(t)\,dt = l$$

$f \in C^0$, ma $f \notin PC^\infty$ perchè la derivata diverge

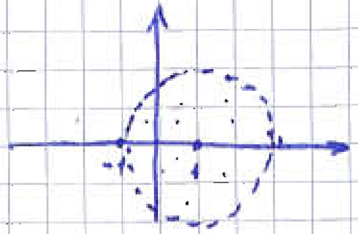
JERK → derivata dell'accelerazione

18/04 1° COMPITINO

8/06 2° COMPITINO

9/06 APPELLO

$$f(s) = \frac{s+3}{(s-1)^2(s+1)}$$



Singolarità in ±1

Scelgo $s_0 = 1$

Nelle funzioni razionali, posso evitare di calcolare gli integral

$$f(s) = \frac{2}{(s-1)^2} - \frac{1/2}{s-1} + \frac{1/2}{s+1}$$

impareremo a fare i conti per arrivare qui

termine corretto ↑ calcolo Taylor

$$D^i\left[\frac{1/2}{s+1}\right] = \frac{1}{2} \cdot (-1)^i \cdot \frac{i!}{(s+1)^{i+1}}$$

$$D^i\left[\frac{1/2}{s+1}\right]_{s=1} = \frac{1}{2}(-1)^i \cdot \frac{i!}{2^{i+1}}$$

TAYLOR $i=0$

$$\frac{1}{2} \cdot \frac{1}{s+1} = \frac{1}{4} - \frac{1}{8}(s-1) + \frac{1}{16}(s-1)^2 \dots$$

$$\frac{s+3}{(s-1)^2(s+1)} = \frac{2}{(s-1)^2} - \frac{1/2}{s-1} + \frac{1}{4} - \frac{1}{8}(s-1) + \frac{1}{16}(s-1)^2 \dots$$

Il residuo è $-\frac{1}{2}$

16/03/09

TRASFORMATA DI LAPLACE

Trasforma l'equazione differenziale in equazione algebrica.

Si applica a funzioni reali $\mathbb{R} \to \mathbb{R}$ $(\mathbb{C})$ per le quali

- $f \in PC^\infty$, cioè derivabile infinite volte eccetto per un numero finito di punti. $Df \in PC^\infty$ $\rightsquigarrow$

$$\int_a^b f(\varphi)\,d\varphi = \int_{a^-}^b f(\varphi)\,d\varphi = \int_{a^+}^b f(\varphi)\,d\varphi$$ sempre ben definito, anche per a, b punti di discontinuità

- $\exists\, \sigma \in \mathbb{R} : \int_0^{+\infty} |f(t)|\, e^{-\sigma t}\, dt < +\infty$, cioè converge

ASCISSA DI CONVERGENZA $\rightarrow$ estremo inferiore dei σ che rendono convergente $\int_0^{+\infty} |f(t)|\, e^{-\sigma t}\, dt$

$$F(s) = \int_{0^-}^{+\infty} f(t)\, e^{-st}\, dt$$ $\forall s \in \mathbb{C}$ per i quali l'integrale converge.

$\uparrow$ variabile complessa

$\uparrow$ dato che $f(t) \in PC^\infty$, uguale a 0 e a 0^+.

$$F(s) = \mathcal{L}\left[f(t)\right]$$

GRADINO UNITARIO

$$1(t) = \begin{cases} 1 & \text{per } t \geq 0 \\ 0 & \text{per } t < 0 \end{cases} \qquad \mathcal{L}\left[1(t)\right] = \int_{0^-}^{+\infty} 1(t)\, e^{-st}\, dt = \int_0^{+\infty} e^{-st}\, dt = \left[\frac{e^{-st}}{-s}\right]_0^{+\infty} = \frac{1}{s}$$

Vero se $e^{-st} \to 0$ per $t \to +\infty$, cioè per $Re(s) > 0$

La continuazione analitica dice che $\mathcal{L}[1(t)] = \frac{1}{s}$ in tutto il piano ad eccezione dei punti di discontinuità.

$t = 0$ è un polo di ordine 1.

SEGNALE ESPONENZIALE $e^{\alpha t}$

$$\mathcal{L}\left[e^{\alpha t}\right] = \int_{0^-}^{+\infty} e^{\alpha t} e^{-st}\, dt = \int_0^{+\infty} e^{(\alpha - s)t}\, dt = \left[\frac{e^{-(s-\alpha)t}}{-(s-\alpha)}\right]_0^{+\infty} = \frac{1}{s-\alpha}$$

Vero se $e^{-(s-\alpha)t} \to 0$ per $t \to +\infty$, cioè quando $Re(s-\alpha) > 0$

$$\mathcal{L}\left[e^{\alpha t}\right] = \frac{1}{s-\alpha} \quad \forall s \in \mathbb{C} - \{\alpha\}$$ per continuazione analitica.

PROPRIETÀ DELLA TRASFORMATA

- LINEARITÀ → $\mathcal{L}\left[c_1 f_1(t) + c_2 f_2(t)\right] = c_1 \mathcal{L}\left[f_1(t)\right] + c_2 \mathcal{L}\left[f_2(t)\right]$

  La trasformata di una costante è la costante per la trasformata del gradino!

- INIETTIVITÀ → $F(s)$ identifica univocamente $f(t)$

  ⇒ è definita la TRASFORMATA INVERSA DI LAPLACE

  Sia $F(s) = \mathcal{L}\left[f(t)\right]$, allora $f(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} F(s)\, e^{st}\, ds \quad \forall \sigma_0 > \sigma_c$

  $f(t) = \mathcal{L}^{-1}\left[F(s)\right]$

- TRASFORMATA DELLA DERIVATA → $\mathcal{L}\left[D f(t)\right] = s F(s) - f(0+)$

- TRASFORMATA DELL'INTEGRALE → $\mathcal{L}\left[\int_0^t f(v)\, dv\right] = \frac{1}{s} F(s)$

TEOREMA DEL VALORE FINALE

Sia $f \in C^1(\mathbb{R}_+)$ con f e Df aventi ascisse di convergenza non positive.
Se esiste il $\lim_{t \to +\infty} f(t)$ vale $\lim_{t \to +\infty} f(t) = \lim_{s \to 0} s \cdot F(s)$

TEOREMA DEL VALORE INIZIALE

Se $\exists \lim_{s \to +\infty} s F(s)$ vale $f(0+) = \lim_{s \to +\infty} s F(s)$

- TRASLAZIONE NEL TEMPO → $\mathcal{L}\left[f(t - t_0)\right] = e^{-t_0 \cdot s} F(s)$

- TRASLAZIONE NELLA VARIABILE COMPLESSA → $\mathcal{L}\left[e^{-\alpha t} f(t)\right] = F(s+\alpha)$

$$\boxed{\mathcal{L}\left[t^n e^{\alpha t}\right] = \frac{n!}{(s-\alpha)^{n+1}}}$$

TEOREMA DI CONVOLUZIONE

$$f * g = \int_0^t f(v)\, g(t-v)\, dv = g * f = \int_0^t g(v)\, f(t-v)\, dv$$

$$\mathcal{L}\left[\int_0^t f(v)\, g(t-v)\, dv\right] = F(s) \cdot G(s)$$

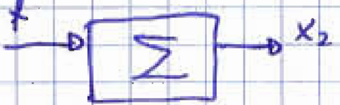
17/03/09

COPPIA DI CARRELLI

Massa concentrata nei carrelli m.

$b \to$ coefficiente di attrito viscoso degli ammortizzatori

Quando entrambe le molle sono a riposo, le rispettive masse si posizionano nell'origine $(x_1 = 0,\ x_2 = 0)$



① $m \cdot D^2 x_1 = f - kx_1 - bDx_1 + k(x_2 - x_1) + b(Dx_2-$

accelerazione

legge di Newton

forza richiamo molla

forza richiamo ammortizzatore

estensione molla

velocità relativa carrello

② $m D^2 x_2 = -k(x_2 - x_1) - b(Dx_2 - Dx_1)$

Metto a sistema ① e ②. La variabile x_1 non mi interessa per cui devo eliminarla.

$$mD^2 x_1 + kx_1 + bDx_1 + kx_1 + bDx_1 = f + kx_2 + bDx_2$$

① $(mD^2 + 2bD + 2k)x_1 = f + bDx_2 + kx_2 \quad \leftarrow (bD+k)$

scambio i due operatori

② $(bD + k)x_1 = mD^2 x_2 + kx_2 + bDx_2 \quad \leftarrow (mD^2 + 2bD + 2k)$

$$\Rightarrow \quad (bD+k)(f + bDx_2 + kx_2) = (mD^2 + 2bD + 2k)(mD^2 x_2 + bDx_2 + kx_2)$$

ho quindi eliminato la variabile x_1.

$$(bD+k)f + (bD+k)^2 x_2 = (mD^2 + 2bD + 2k)(mD^2 + bD + k)\, x_2$$

$$(bD+k)f + (bD+k)^2 x_2 = (m^2D^4 + mbD^3 + mD^2k + 2bmD^3 + 2b^2D^2 + 2bkD + 2kmD^2 + 2kbD + 2k^2)$$

$$(bD+k)f + (b^2D^2 + k^2 + 2bkD)\, x_2 = (m^2D^4 + 3mbD^3 + (3km+2b^2)D^2 + 4kbD + 2k^2)\, x_2$$

$$(bD+k)f = (m^2D^4 + 3mbD^3 + (3km+b^2)D^2 + 2kbD + k^2)\, x^2$$

$n = 4$ ← grado

$m = 1$

$\rho \triangleq n-m = 3$

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad n \to n-1 \qquad \mathcal{L}[t^{n-1}] = \frac{(n-1)!}{s^n}$$ divido per $(n-1)!$

$$\mathcal{L}\left[\frac{1}{(n-1)!}t^{n-1}\right] = \frac{1}{s^n}$$ traslo $$\mathcal{L}\left[\frac{1}{(n-1)!}t^{n-1}e^{\alpha t}\right] = \frac{1}{(s-\alpha)^n}$$

$$F(s) = \frac{4s^3+2s+9}{(s+2)^4(s+5)^2(s+7)}$$ trovare l'antitrasformata $f(t) = \mathcal{L}^{-1}[F(s)] = ?$

← $n = 7 \Rightarrow 7$ termini

$$f(t) = c_1 e^{-7t} + c_2 e^{-5t} + c_3 t e^{-5t} + c_4 e^{-2t} + c_5 t e^{-2t} + c_6 t^2 e^{-2t} + c_7 t^3 e^{-2t}$$

calcolabili con K_{ij}

POLINOMI COPRIMI TRA LORO → non hanno radici in comune

ESERCIZI – SLIDE 14

$$F_1(s) = \frac{{}^{\boxed{*}}4s+1}{(s+1)(s+5)} = \frac{K_1}{s+1} + \frac{K_2}{s+5} = -\frac{3}{4}\cdot\frac{1}{s+1} + \frac{19}{4}\cdot\frac{1}{s+5}$$

$m=1$, $n=2 \Rightarrow \rho = 1$

$\Downarrow$

$K_1 + K_2 = 4^{\boxed{*}}$

$$K_1 = \cancel{(s+1)}\cdot\frac{4s+1}{\cancel{(s+1)}(s+5)}\bigg|_{s=-1} = -\frac{3}{4} \qquad K_2 = \cancel{(s+5)}\frac{4s+1}{(s+1)\cancel{(s+5)}}\bigg|_{s=-5} = \frac{19}{4}$$

$$f_1(t) = \mathcal{L}^{-1}[F_1(s)] = -\frac{3}{4}\cdot e^{-t} + \frac{19}{4}\cdot e^{-5t}$$ per $t \geq 0$

grado 1

$$F_2(s) = \frac{1}{s(s+1)^3(s+2)} = \frac{K_1}{s} + \frac{K_{21}}{(s+1)^3} + \frac{K_{22}}{(s+1)^2} + \frac{K_{23}}{(s+1)} + \frac{K_3}{s+2}$$

$$K_1 = \cancel{s} \cdot \frac{1}{\cancel{s}(s+1)^3(s+2)}\Bigg|_{s=0} = \frac{1}{2} \qquad K_{21} = \cancel{(s+1)^3} \cdot \frac{1}{s\cancel{(s+1)^3}(s+2)}\Bigg|_{s=-1} = -1$$

$$K_3 = \cancel{(s+2)} \cdot \frac{1}{s(s+1)^3\cancel{(s+2)}}\Bigg|_{s=-2} = \frac{1}{2} \qquad K_{22} = D\left[\frac{1}{s(s+2)}\right]\Bigg|_{s=-1} = \frac{-2s-2}{s^2(s+2)^2}\Bigg|_{s=-1} = 0$$

$$K_{23} = \frac{D^2}{2!}\left[\frac{1}{s(s+2)}\right]\Bigg|_{s=-1} = \ldots$$ uso invece le proprietà dei residui

$m=0$
$n=5$ $\quad \rho = 5 \Rightarrow \sum R_i = 0 \Rightarrow K_1 + K_{23} + K_3 = 0 \Rightarrow K_{23} = -1$

$$F_2(s) = \frac{1}{2} \cdot \frac{1}{s} - \frac{1}{(s+1)^3} - \frac{1}{s+1} + \frac{1}{2} \cdot \frac{1}{s+2}$$

$$f_2(t) = \mathcal{L}^{-1}\left[F_2(s)\right] = \frac{1}{2} - \frac{1}{2}t^2 e^{-t} - e^{-t} + \frac{1}{2}e^{-2t} \quad \text{per } t \geq 0$$

19/03/09

$$\mathcal{L}\left[Df\right] = s \cdot F(s) - f(0+)$$

SOLUZIONI FORTI → soluzioni derivabili fino all'ordine necessario

SOLUZIONI DEBOLI → funzioni discontinue, soluzioni dell'equazione differ

26/03/09

$$F_3(s) = \frac{64}{(s+2)(s^2+4)} = \frac{64}{(s+2)(s-j2)(s+j2)} = \frac{K_1}{s+2} + \frac{K_2}{s-j2} + \frac{\bar{K}_2}{s+j2}$$

$$\mathcal{L}^{-1}\left[\frac{K}{s-p} + \frac{\bar{K}}{s-\bar{p}}\right] = 2|K| e^{\mathrm{Re}\,p \cdot t} \cos\left[\mathrm{Im}\,p \cdot t + \arg K\right]$$

$$K_1 = \left.\frac{64}{(s^2+4)}\right|_{s=-2} = 8$$

$$K_2 = \left.\frac{64}{(s+2)(s+j2)}\right|_{s=j2} = \frac{64}{(2+j2)\cdot 4j} = \frac{64}{8j-8} = \frac{8}{j-1}\cdot\frac{j+1}{j+1} = \frac{8j+8}{-1-1} = -4-4j$$

$$\overline{K_2} = -4+4j$$

$$F_3(s) = \frac{8}{s+2} - \frac{4+4j}{s-j2} + \frac{-4+4j}{s+j2}$$

$$f_3(t) = \mathcal{L}^{-1}\left[F_3(s)\right] = 8e^{-2t} + 2\cdot 4\sqrt{2}\cdot e^{\alpha t}\cdot\cos\left[2t-\frac{\pi}{2}-\right.$$

$$= 8e^{-2t} + 8\sqrt{2}\ \mathrm{sen}\left(2t-\frac{\pi}{4}\right) \text{ per } t\geq 0$$

$$\arg K = \mathrm{arctg}\,\frac{-4}{-4}$$

$$\mathcal{L}\left[\mathrm{sen}\,\omega t\right] = ? \qquad \mathcal{L}\left[\cos\omega t\right] = ?$$

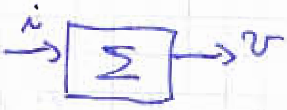
$$\begin{cases} e^{j\omega t} = \cos\omega t + j\,\mathrm{sen}\,\omega t \\ e^{-j\omega t} = \cos\omega t - j\,\mathrm{sen}\,\omega t \end{cases} \quad \begin{matrix}\text{SOMMO}\\ \text{SOTTRAGGO}\end{matrix} \begin{cases} 2\cos\omega t = e^{j\omega t} + e^{-j\omega t} \\ 2j\,\mathrm{sen}\,\omega t = e^{j\omega t} - e^{-j\omega t} \end{cases} \quad \begin{cases} \cos\omega t = \frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) \\ \mathrm{sen}\,\omega t = \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right) \end{cases}$$

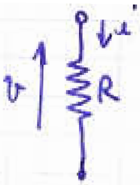
$$\mathcal{L}\left[\cos\omega t\right] = \frac{1}{2}\left(\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right) = \frac{1}{2}\,\frac{s+j\omega+s-j\omega}{(s-j\omega)(s+j\omega)} = \frac{s}{s^2+\omega^2}$$

$$\mathcal{L}\left[\mathrm{sen}\,\omega t\right] = \frac{1}{2j}\left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right) = \frac{1}{2j}\left(\frac{\cancel{s}+j\omega-\cancel{s}+j\omega}{(s-j\omega)(s+j\omega)}\right) = \frac{\omega}{s^2+\omega^2}$$

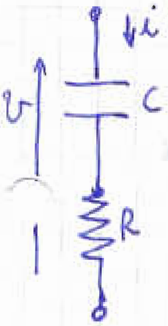
$$\boxed{\begin{aligned} \mathcal{L}\left[\cos\omega t\right] &= \frac{s}{s^2+\omega^2} \\ \mathcal{L}\left[\mathrm{sen}\,\omega t\right] &= \frac{\omega}{s^2+\omega^2} \end{aligned}}$$

IMPEDENZA → funzione di trasferimento del sistema (bipolo) orientato dalla corrente alla tensione





$Z(s) = R$ perché $v = R \cdot I \xrightarrow{\mathcal{L}} V(s) = R \cdot I(s)$



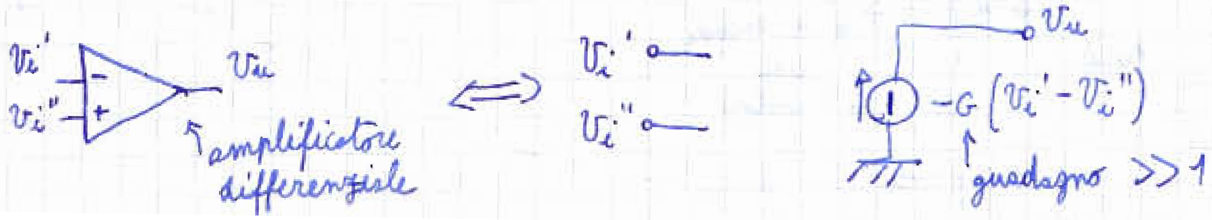
$$v = R \cdot I + \frac{1}{C}\int_{-\infty}^{t} i(\xi)\, d\xi \qquad Dv = R \cdot Di + \frac{1}{C} \xrightarrow{\mathcal{L}} sV(s) = R \cdot sI(s) + \frac{1}{C} I(s)$$

$$V(s) = \left(R + \frac{1}{sC}\right) I(s) \qquad Z(s) = R + \frac{1}{sC}$$

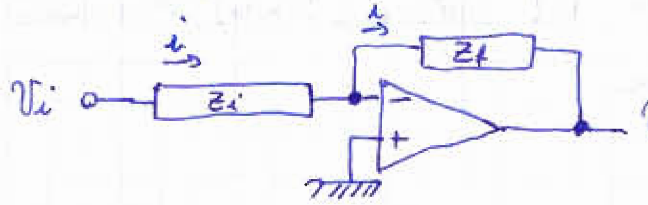
L'impedenza della capacità (R=0), sarebbe : $1/sC$

$Z(s) = R + sL$

AMPLIFICATORE OPERAZIONALE → dispositivo che permette di fare operazioni su segnali elettrici



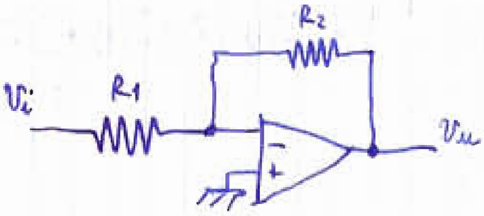
SCHEMA LOGICO AMPLIFICATORE OPERAZIONALE



Z_f → feedback
Z_i → ingresso } → impedenze

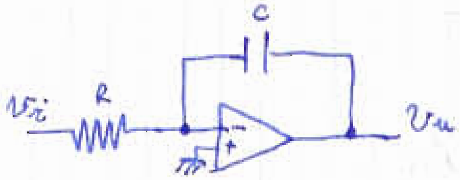
PRINCIPIO DI MASSA VIRTUALE → se G molto elevato e V_u ha un valore finito, la V_i la considero virtualmente a massa

$$I(s) = \frac{V_i(s)}{Z_i(s)} = -\frac{V_u(s)}{Z_f(s)} \qquad \underset{\text{RISPOSTA FORZATA}}{V_u(s)} = -\underset{\text{GUADAGNO}}{\frac{Z_f(s)}{Z_i(s)}} \cdot V_i(s)$$



$$G(s) = -\frac{R_2}{R_1}$$

AMPLIFICATORE INVERTENTE

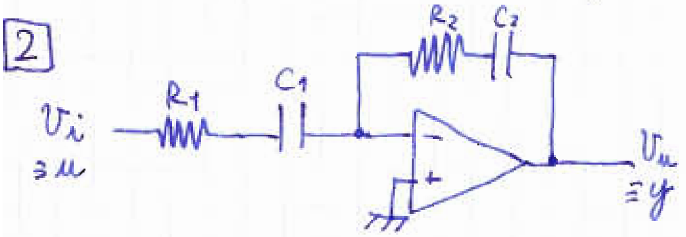


$$G(s) = -\frac{\frac{1}{SC}}{R} = -\frac{1}{RC \cdot S}$$

CIRCUITO INTEGRATORE

$$V_u(s) = -\frac{1}{RCS} V_i(s) = -\frac{1}{RC}\left\{\frac{1}{S} \cdot V_i(s)\right\} \xRightarrow[\mathcal{L}^{-1}]{} V_u(t) = -\frac{1}{RC} \cdot \int_0^t V_i(\varphi)\, d\varphi$$

$\boxed{2}$

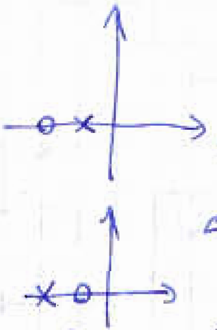


$$G(s) = \frac{R_2 + \frac{1}{SC_2}}{R_1 + \frac{1}{SC_1}} = \frac{\frac{1+R_2C_2S}{\cancel{S}C_2}}{\frac{1+R_1C_1S}{\cancel{S}C_1}} = -\frac{C_1}{C_2} \cdot \frac{1+R_2C_2S}{1+R_1C_1S} = \frac{-C_1 - R_2C_1C_2S}{C_2 + R_1C_1C_2S}$$

coefficienti sviluppo y

$$Z_1 = -\frac{1}{R_2C_2} \quad \text{ZERO}$$

$$P_1 = -\frac{1}{R_1C_1} \quad \text{POLO}$$

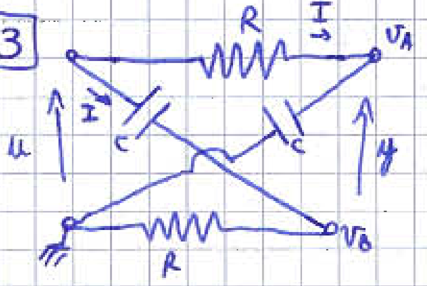


a seconda dei valori

$$R_1C_1C_2\, Dy + C_2\, y = -R_2C_1C_2\, Du - C_1\, u$$

$\boxed{3}$

30/03/09



$$Y(s) = G(s)\,U(s)$$

$$y = V_A - V_B$$

I è la stessa perchè prima di arrivare a massa si incontrano sempre una resistenza e un condensatore

$$I(s) = \frac{U(s)}{R + \frac{1}{sC}} \qquad V_A = \frac{1}{sC} I(s) \qquad V_B = R \cdot I(s) \qquad y = V_A - V_B$$

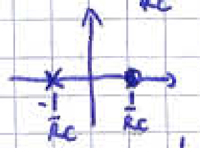
$$Y(s) = \left(\frac{1}{sC} - R\right) I(s) = \left(\frac{1}{sC} - R\right) \cdot \frac{U(s)}{R + \frac{1}{sC}} = \frac{\frac{1-sRC}{\cancel{sC}}}{\frac{1+sRC}{\cancel{sC}}} U(s) = \underbrace{\frac{1-sRC}{1+sRC}}_{G(s)} U(s)$$

$$Y(s)\,(1+sRC) = (1-sRC)\,U(s)$$

$$Y + RCsY = U - RCsU \xrightarrow{s \to D} y(t) + RC\,Dy(t) = u(t) - RC\,Du(t)$$

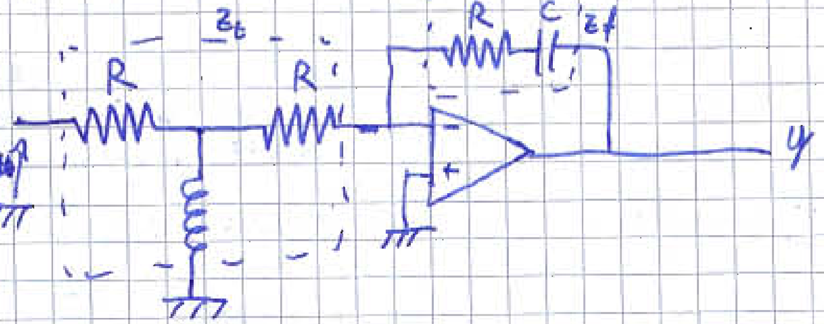
POLO $-\frac{1}{RC}$

ZERO $\frac{1}{RC}$



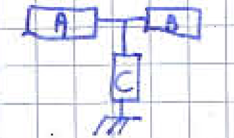
MODO: $e^{-\frac{1}{RC}t}$

$y_{LIBERA} = c\,e^{-\frac{1}{RC}t}$



$$T(s) = \frac{-Zf}{Z_t} = \frac{-\left(R + \frac{1}{sC}\right)}{R + R + \frac{R \cdot R}{sL}} \Longrightarrow -\frac{s + \frac{1}{RC}}{2\left(s + \frac{R}{2L}\right)} \qquad y = TU$$

← formula



$$A + B + \frac{AB}{C}$$

$$2sT + \frac{R}{L}T = -s - \frac{1}{RC} \qquad 2Dy + \frac{R}{L}y = -Du - \frac{1}{RC}u$$

$$T(0) = -\frac{1}{RC} \cdot \frac{L}{R} = -\frac{L}{R^2C}$$

$$\underset{\text{polo}}{p} = -\frac{R}{2L} \longrightarrow e^{-\frac{R}{2L}t} \qquad \text{GUADAGNO STATICO} = -\frac{L}{R^2C}\, e^{-\frac{R}{2L}t}$$

$$d(t) = 7\delta(t) + 4\delta^{(1)}(t) + 32\,\delta(t-10) + 2\,\delta^{(2)}(t-10)$$

$$f(t) = 1(t) + 2t \cdot 1(t-5)$$

$$D^* f(t) = \delta(t) + 2 \cdot 1(t-5) + \underbrace{2t\,\delta(t-5)}_{10\,\delta(t-5)}$$

$$D^{*2} f(t) = \delta^{(1)}(t) + 2\delta(t-5) + 10\,\delta^{(1)}(t-5) \in I^*$$

$\boxed{6}$

$u(t) = 1(t)$

$$G(s) = \frac{s-2}{(s+2)^3(s+1)}$$

$$Y(s) = G(s)\,U(s) = \frac{s-2}{(s+2)^3(s+1)} \cdot \frac{1}{s} = \frac{K_1}{s} + \frac{K_2}{(s+2)^3} + \frac{K_3}{(s+2)^2} + \frac{K_4}{s+2} + \frac{K_5}{s+1}$$

$$K_1 = s\,Y(s)\Big|_{s=0} = \frac{-2}{8 \cdot 1} = -\frac{1}{4}$$

RESIDUI POLI DISTINTI

$$\begin{cases} \text{se } \rho > 1 & \sum_{1}^{n} R_i = 0 \\ \text{se } \rho = 1 & \sum_{1}^{n} R_i = b_n \end{cases}$$

$$K_2 = (s+2)^3\,Y(s)\Big|_{s=-2} = -2 \qquad K_5 = (s+1)\,Y(s)\Big|_{s=-1} = 3$$

$$K_1 + K_4 + K_5 = 0 \qquad K_4 = -\frac{11}{4}$$

$$K_3 = \frac{d}{ds}\left[(s+2)^3 \cdot Y(s)\right]_{s=-2} = -\frac{5}{2}$$

$$\frac{1}{2} D^2\left[\frac{s-2}{s(s+1)}\right]\Bigg|_{s=-2}$$

$$y(t) = -\frac{1}{4} - 2\,\mathcal{L}^{-1}\left[\frac{1}{(s+2)^3}\right] - \frac{5}{2}\,\mathcal{L}^{-1}\left[\frac{1}{(s+2)^2}\right] - \frac{11}{4}e^{-2t} + 3e^{-t} = -\frac{1}{4} - \cancel{2} \cdot \frac{1}{\cancel{2}} t^2 e^{-2t} - \frac{5}{2} t e^{-2t} - \frac{11}{4} e^{-2t} + 3e^{-t}$$

31/03/09

$\boxed{4}$

$a_2 D^2 y + a_1 D y + a_0 y = b_2 D^2 u + b_1 D u + b_0 u$ sistema di 2° ordine

grado relativo $= \rho = n - m = 0$

$$\begin{bmatrix} a_n & & 0 \\ a_{n-1} & \ddots & \\ \vdots & \ddots & \ddots \\ & \cdots & a_n \end{bmatrix} \begin{bmatrix} y_+ - y_- \\ Dy_+ - Dy_- \\ \vdots \\ D^n y_+ - D^n y_- \end{bmatrix} = \begin{bmatrix} b_n & & 0 \\ \vdots & \ddots & \\ & \ddots & b_n \end{bmatrix} \begin{bmatrix} u_+ - u_- \\ \vdots \\ D^n u_+ - D^n u_- \end{bmatrix}$$

$$\begin{bmatrix} a_2 & 0 \\ a_1 & a_2 \end{bmatrix} \begin{bmatrix} y_+ - y_- \\ Dy_+ - Dy_- \end{bmatrix} = \begin{bmatrix} b_2 & 0 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} u_+ - u_- \\ Du_+ - Du_- \end{bmatrix}$$ Devo dimostrare questo risultato:

$a_2 D^{*2} y + a_1 D^{*} y + a_0 y = b_2 D^{*2} u + b_1 D^{*} u + b_0 u$ suppongo ci sia una discontinuità solo per t

$$a_2\left(D^2 y + (y_+ - y_-)\delta^{(1)}(t) + (Dy_+ - Dy_-)\delta(t)\right) + a_1\left(Dy + (y_+ - y_-)\delta(t)\right) + a_0 y =$$

$$= b_2\left(D^2 u + (u_+ - u_-)\delta^{(1)}(t) + (Du_+ - Du_-)\delta(t)\right) + b_1\left(Du + (u_+ - u_-)\delta(t)\right) + b_0 u$$

valido per $t=0$: D^2y è trascurabile rispetto ai termini impulsivi

$$\begin{cases} a_2(y_+ - y_-) = b_2(u_+ - u_-) \\ a_2(Dy_+ - Dy_-) + a_1(y_+ - y_-) = b_2(Du_+ - Du_-) + b_1(u_+ - u_-) \end{cases}$$

I coefficienti associati a $\delta(0)$ devono essere uguali per soddisfare l'equazioni. Il sistema equivale alla forma matriciale scritta sopra

$\boxed{5}$

$4D^2y + 2Dy + 7y = 3Du + u$ noti $u(t)$ per $t \geq 0$, e le condizioni iniziali $Dy(0-)$, $y(0-)$ e $u(0-)$, trovare $Y(s)$

Considero l'e.d. con le derivate generalizzate perché conosco le condizioni iniziali a 0^- (se fossero a 0^+ userei la normale)

$$4\left(s^2 Y(s) - y_- s - Dy_-\right) + 2\left(sY(s) - y_-\right) + 7Y(s) = 3\left(sU(s) - u_-\right) + U(s)$$

$$(4s^2 + 2s + 7)Y(s) - 4y_- s - 4Dy_- - 2y_- = (3s+1)U(s) - 3u_-$$

$$Y(s) = \frac{3s+1}{\underset{\uparrow}{4s^2+2s+7}} U(s) + \frac{4y_- s + 4Dy_- + 2y_- - 3u_-}{\underset{\uparrow}{4s^2+2s+7}}$$

$y(t) =$ RISPOSTA FORZATA + RISPOSTA LIBERA

$\boxed{6}$ *

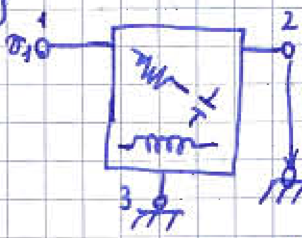
$y(t) \in \overline{C^{\rho-1,\infty}}$ $\quad \underset{\uparrow}{\rho} = 4-1 = 3$ $\quad y(t) \in \overline{C^{3,\infty}}$ $\quad$ GRADO MAX $= 2$

FUNZIONE DI TRASFERIMENTO

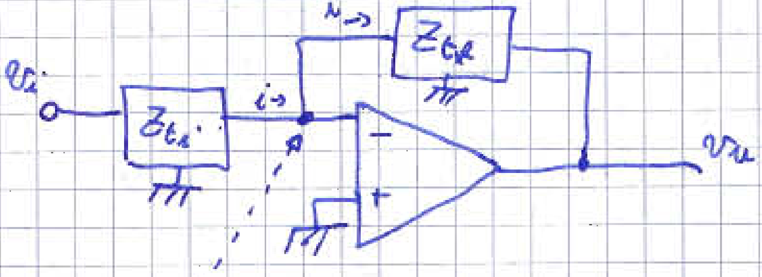
Si potrebbe verificare vedendo se $Dy(0-) = Dy(0+)$, $D^2y(0-) = D^2y(0+)$...

ESERCITAZIONE 3

$\boxed{1}$



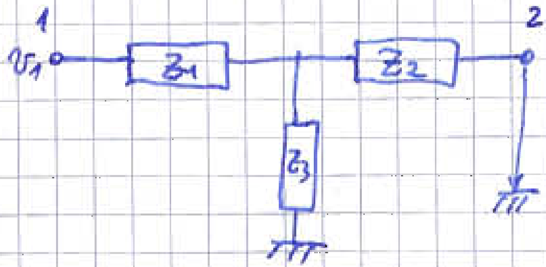
$$Z \triangleq \frac{V_1(s)}{I_{2c}(s)} \Bigg|_{\text{cond. iniziali nulle}}$$



virtualmente a massa

$$G(s) = -\frac{Z_{tf}}{Z_{ti}} \qquad I(s) = \frac{V_i(s)}{Z_{ti}(s)}$$

$$I(s) = -\frac{V_u(s)}{Z_{tf}(s)}$$

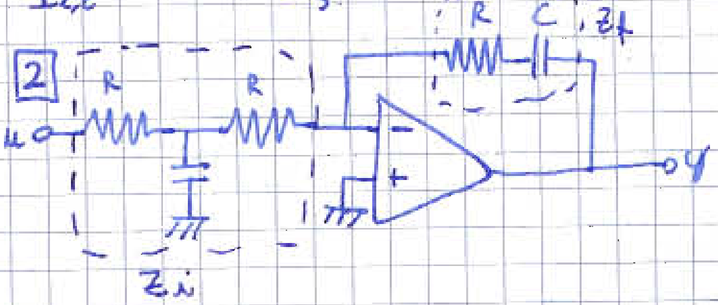


IMPEDENZA DI TRASFERIMENTO $= Z_1 + Z_2 + \frac{Z_1 \cdot Z_2}{Z_3}$

$$I = \frac{V_1}{Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}} \qquad I_{2c} = I \cdot \frac{Z_3}{Z_2 + Z_3} = \frac{V_1}{Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}} \cdot \frac{Z_3}{Z_2 + Z_3} = V_1 \cdot \frac{Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$\frac{V_1}{I_{2,c}} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3} = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$ ▨

$\boxed{2}$



$$G(s) = -\frac{R + \frac{1}{sC}}{2R + R^2 sC} = -\frac{\frac{RsC+1}{sC}}{2R + R^2 sC} =$$

$$= \frac{-(RsC+1)}{2RsC + R^2 s^2 C^2}$$

$$Y(s) = G(s)\,U(s) = \frac{-(RsC+1)}{2RsC + R^2 s^2 C^2}\,U(s) \qquad 2RsC\,Y(s) + R^2 s^2 C^2\,Y(s) = -(RsC+1)\,U(s)$$

$$R^2C^2D^2y(t) + 2RCDy(t) = -RCDu(t) - u(t)$$

Poli: $0, -\frac{2}{RC}$ modi $= \left\{e^{0\cdot t}, e^{-\frac{2}{RC}t}\right\} = \left\{1, e^{-\frac{2}{RC}t}\right\}$

Zeri: $-\frac{1}{RC}$

06/04/09

$g(t) = 15e^{-2t} - 10te^{-2t} - 15e^{-4t}$ trovare la risposta al gradino $g_s(t)$.

1] $g_s(t) = \int_0^t g(\tau)\,d\tau$ 2] oppure $\mathcal{L}^{-1}\left[G_s(s) = \frac{1}{s}G(s)\right]$

1] $$g_s(t) = \int_0^t \left(15e^{-2t} - 10te^{-2t} - 15e^{-4\tau}\right)d\tau = 15\int_0^t e^{-2\tau}d\tau - 10\int_0^t \tau e^{-2\tau}d\tau - 15\int_0^0 e^{-4\tau}d\tau =$$

$$= -\frac{15}{2}\left[e^{-2t} - 1\right] - 10\left[-\frac{1}{2}te^{-2t} - \frac{1}{4}e^{-2t} + \frac{1}{4}\right] - 15\left[\frac{1}{4}\left(e^{-4t} - 1\right)\right] =$$

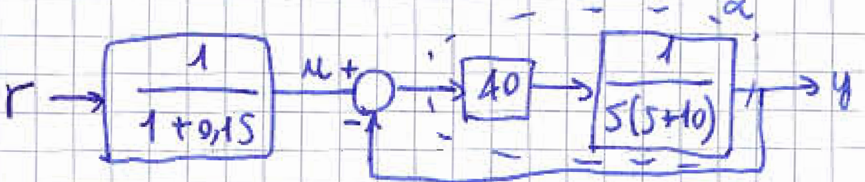
$$= \frac{5}{4} - 5e^{-2t} + 5te^{-2t} + \frac{15}{4}e^{-4t}$$

2] $$G_s(s) = \frac{1}{s}G(s) = \frac{1}{s}\left[\frac{15}{s+2} - \frac{10}{(s+2)^2} - \frac{15}{(s+4)}\right] = \frac{(s+2)^2(s+4)\left[15s(s+2)(s+4) - 10s(s+4) - 15s(s\right.}{s(s+2)^2(s+4)}$$

$$= \dots = \frac{20(s+1)}{s(s+2)^2(s+4)} = \frac{K_1}{s} + \frac{K_2}{(s+2)^2} + \frac{K_3}{(s+2)} + \frac{K_4}{s+4}$$

$$K_1 = \left.\frac{20(s+1)}{(s+2)^2(s+4)}\right|_{s=0} = \frac{20}{16} = \frac{5}{4} \qquad K_2 = \left.\frac{20(s+1)}{s(s+4)}\right|_{s=-2} = 5$$

$$K_4 = \left.\frac{20(s+1)}{s(s+2)^2}\right|_{s=-4} = \frac{15}{4} \qquad K_3 = -5$$



1) calcolo della funzione di trasferimento

$$T_{uy} = \frac{\frac{40}{S(S+10)}}{1+\frac{40}{S(S+10)}} = \frac{40}{S^2+10S+40}$$

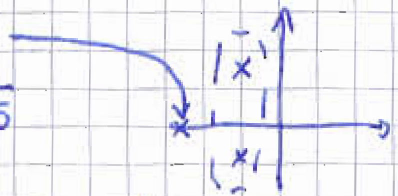
$\frac{\alpha}{1+\alpha}$

lezione 1

$$T_{ry} = \frac{1}{1+0,1S} \cdot \frac{40}{S^2+10S+40} = \frac{40}{(1+0,1S)(S^2+10S+40)}$$

POLI: $p_1 = -10$

$p_{2,3} = -5 \pm j\sqrt{15}$



APPROSSIMO CON I POLI DOMINANTI

$$T_{ry} = \frac{40}{(S^2+10S+40)} = \frac{\omega_m^2}{S^2+2\delta\omega_m S+\omega_m^2} \Rightarrow$$

$\omega_m = \sqrt{40} = 2\sqrt{10}\ \frac{rad}{s}$

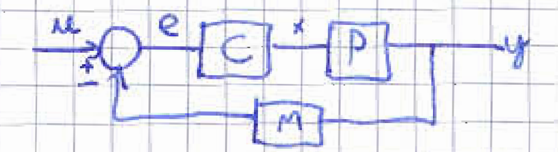
$\delta = \frac{10S}{2\omega_m} = 0,79$

$$T_a = \frac{-3}{\delta\omega_m} = 0,6S \qquad S = 100e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} = 1,7\%$$

$e = u - y$

$y = PC \cdot e = PC(u-y) = PCu - PCy$

$$(1+PC)y = PCu \qquad y = \frac{PC}{1+PC}u$$



$e = u - My$

$y = PCe = PC(u-My) = PCu - PCMy$

$$(1+PCM)y = PCu \qquad y = \frac{PC}{1+PCM}y$$

$g(s) = \frac{1-s}{(s+1)(s+2)}$ $\quad Y = \frac{1}{s^2} \cdot \frac{1-s}{(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s+2}$

$u(t) = t \cdot 1(t)$

$y(t) = ?$ $\quad B = \frac{1}{2} \quad C = 2 \quad D = -\frac{3}{4} \quad A = \frac{d}{ds}\left[\frac{1-s}{(s+1)(s+2)}\right]_{s=0} = -\frac{5}{4}$

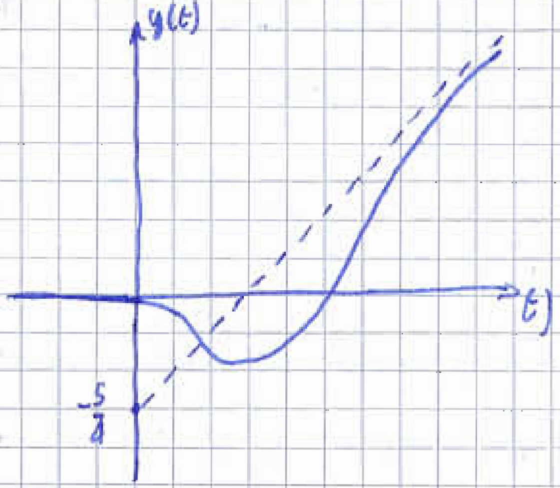
grado continuità?

grafico? $\quad y(t) = -\frac{5}{4} + \frac{1}{2}t + 2e^{-t} - \frac{3}{4}e^{-2t}$ per $t \geq 0$

$u(t) \in C^0$ perché non è derivabile in 0.

grado di continuità di $G = \underset{\text{den}}{2} - \underset{\text{num}}{1} = 1$

grado di continuità di $Y = 0 + 1 = 1 \quad y \in C^{1,\infty}$



$D^2 y = 2e^{-t} - 3e^{-2t} \quad t \geq 0$

$D^2 y(0) = -1 < 0$ concavità bassa

ESERCITAZIONI: Ing. Gabriele Lini

TUTORATO: Ing. Riccardo Pecori

ESERCITAZIONE 3

3) $g_s(t) = \frac{1}{2} + e^{-t} - \frac{3}{2}e^{-2t}$

Dall'integrale di V.

$$y(t) = \int_0^t u'(v)\, g_s(t-v)\, dv + u(0^+) \cdot g_s(t)$$

$y(t) = ?$

oppure uso la trasformata di Laplace

$$u(t) = \begin{cases} 0 & \text{per } t<0 \\ 1+t & \text{per } t \geq 0 \end{cases}$$

- $Y(s) = G(s) \cdot U(s) \quad g_s(t) = \mathcal{L}^{-1}\left[G(s) \cdot \frac{1}{s}\right]$

$G(s) = s \cdot \mathcal{L}[g_s(t)]$

Un altro modo è: $(1(t), g_s(t)) \in B \Rightarrow D^*(1(t), g_s(t)) \in B^* \Rightarrow (\delta(t), \underbrace{D^* g_s(t)}_{g(t)}) \in B^*$

$g(t) \rightarrow$ risposta all'impulso

$\boxed{g(t) = D^* g_s(t)}$ se $g_s(t)$ è continua $g(t) = D g_s(t)$

se $g_s(0^-) = g_s(0^+)$ (come in questo caso), $g_s(t)$ è continua

$$g(t) = D g_s(t) = -e^{-t} + 3e^{-2t} \qquad G(s) = \mathcal{L}[g(t)] = -\frac{1}{s+1} + 3\frac{1}{s+2}$$

$$U(s) = \mathcal{L}[u(t)] = \frac{1}{s} + \frac{1}{s^2}$$

$$Y(s) = \left(-\frac{1}{s+1} + \frac{3}{s+2}\right)\left(\frac{1}{s} + \frac{1}{s^2}\right) = \frac{-s-2+3s+3}{\cancel{(s+1)}(s+2)} \cdot \frac{\cancel{s+1}}{s^2} = \frac{2s+1}{s^2(s+2)} = \frac{K_{11}}{s^2} + \frac{K_{12}}{s} + \frac{K_2}{s+2}$$

$$K_{11} = \left.\frac{2s+1}{s+2}\right|_{s=0} = \frac{1}{2} \qquad K_2 = \left.\frac{2s+1}{s^2}\right|_{s=-2} = -\frac{3}{4}$$

Ci sono due poli, $s=0$ e $s=-2$. I coefficienti associati ai poli sono rispettivamente K_{12} e K_2. Dato che ρ (differenza gradi) $= 3-1 = 2$ ed è maggiore di 1, la somma dei residui deve dare 0. Altrimenti deve essere uguale al coefficiente del termine di grado più alto a numeratore.

$$K_{12} + K_2 = 0 \Rightarrow K_{12} = -K_2 = \frac{3}{4}$$

$$y(t) = \frac{1}{2}t + \frac{3}{4} - \frac{3}{4}e^{-2t}$$

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a}$$

$D^2 y + 4Dy + 4y = D^2 u + 2Du + u$

per $t<0$ $u(t)=2e^{-t}$, $y(t)=e^{-2t}$

$u(t)=10$ per $t \geq 0$

- verificare che le condizioni iniziali soddisfino l'equazione

$D(u(t)) = -2e^{-t}$ $\qquad D(y(t)) = -2e^{-2t}$

$D^2(u(t)) = 2e^{-t}$ $\qquad D^2(y(t)) = 4e^{-2t}$

$$4e^{-2t} + 4\cdot(-2e^{-2t}) + 4e^{-2t} = 2e^{-t} - 4e^{-t} + 2e^{-t} \Rightarrow 0=0 \quad \checkmark$$

Dato che devo analizzare il sistema per ogni t e che le condizioni iniziali non sono nulle, uso le derivate generalizzate.

- condizioni iniziali: fino a $n-1$ per uscita e $m-1$ per ingresso

$u(0^-) = 2$ $\qquad y(0^-) = 1$

$Du(0^-) = -2$ $\qquad Dy(0^-) = -2$

- trasformo con Laplace

$$s^2 Y(s) - s y(0^-) - Dy(0^-) + 4\left[sY(s) - y(0^-)\right] + 4Y(s) = s^2 U(s) - s u(0^-) - Du(0^-) + 2\left[sU(s) - u(0^-)\right]$$

$$s^2 Y(s) - s - 2 + 4sY(s) - 4 + 4Y(s) = s^2 U(s) - 2s + 2 + 2sU(s) - 4 + U(s)$$

$$Y(s) = \underbrace{\frac{s^2+2s+1}{s^2+4s+4}U(s)}_{\text{RISPOSTA FORZATA}} - \underbrace{\frac{s}{s^2+4s+4}}_{\text{RISPOSTA LIBERA}} \qquad U(s) = \frac{10}{s}$$

nel caso, si può antitrasformare singolarmente le due risp arrivando allo stesso risulta

$$Y(s) = \frac{(s^2+2s+1)\cdot 10}{s(s^2+4s+4)} - \frac{s}{s^2+4s+4} = \frac{10s^2+20s+10-s^2}{s(s^2+4s+4)} = \frac{9s^2+20s+10}{s(s+2)^2} =$$

$$= \frac{k_1}{s} + \frac{k_{21}}{(s+2)^2} + \frac{k_{22}}{(s+2)} \qquad k_1 = \left.\frac{9s^2+20s+10}{(s+2)^2}\right|_{s=0} = \frac{5}{2} \qquad k_{21} = \left.\frac{9s^2+20s+10}{s}\right|_{s=-2} = -3$$

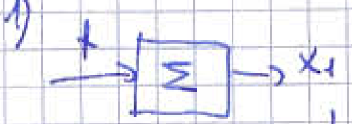
$\rho = 3 - 2 = 1$ $\qquad K_1 + K_{22} = 9$ $\qquad K_{22} = -\frac{5}{2} + 9 = \frac{13}{2}$

$$Y(s) = \frac{5}{2} \cdot \frac{1}{s} - \frac{3}{(s+2)^2} + \frac{13}{2} \cdot \frac{1}{s+2}$$

$$y(t) = \frac{5}{2} - 3te^{-2t} + \frac{13}{2}e^{-2t} \quad \text{per } t \geq 0$$

ESERCITAZIONE 4

1)



masse ammortizzatori e molla trascurabili.

$$\begin{cases} m D^2 x_1 = -b D x_1 + K(x_2 - x_1) \\ m D^2 x_2 = -K(x_2 - x_1) - b D(x_2) + f \end{cases}$$

$(x_2 - x_1)$ estensione molla

Kx_1 deve sempre avere segno − nell'equazione della 1° massa

Suppongo che x_1 sia 0 e $x_2 > 0$, considerando m_1. La molla tende a tirare verso destra m_1, quindi ha segno +.

Scrivo a sinistra i termini che dipendono da x_2

$$\begin{cases} kx_2 = m D^2 x_1 + b D x_1 + k x_1 \\ (mD^2 + bD + k)\, x_2 = f + k x_1 \end{cases} \qquad \begin{cases} k(mD^2 + bD + k) x_2 = (mD^2 + bD + k)(mD^2 x_1 + bD x_1 + kx_1) \\ k(mD^2 + bD + k) x_2 = kf + k^2 x_1 \end{cases}$$

$$m^2 D^4 x_1 + mbD^3 x_1 + mkD^2 x_1 + mbD^3 x_1 + b^2 D^2 x_1 + bkDx_1 + mkD^2 x_1 + kbDx_1 + \cancel{k^2 x_1} = kf + \cancel{k^2 x_1}$$

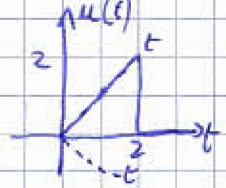
$$m^2 D^4 x_1 + 2mbD^3 x_1 + (2mk + b^2) D^2 x_1 + 2kbDx_1 = kf$$

$$G(s) = \frac{k}{m^2 s^4 + 2mbs^3 + (2mk + b^2)s^2 + 2kbs} = \frac{k}{s(m^2 s^3 + 2mbs^2 + (2mk + b^2)s + 2kb)}$$

$G(0) = +\infty$ perché è un sistema fluttuante che applicando $f = \text{cost}$ non si ferma più finché non arriva a fine corsa degli ammortizzatori

GUADAGNO STATICO

② $G(s) = \dfrac{10}{s+3}$



$y(t) = ?$ $\quad g(t) = \mathcal{L}^{-1}[G(s)] = 10e^{-3t}$ $\quad Y(s) = G(s)U(s)$

$u(t) = t \cdot 1(t) - t\,1(t-2) = t \cdot 1(t) - (t-2+2) \cdot 1(t-2)$ $\quad \mathcal{L}[f(t-t_0)] = e^{-t_0 s}F(s)$

$U(s) = \dfrac{1}{s^2} - e^{-2s}\left(\dfrac{1}{s^2} + \dfrac{2}{s}\right)$ l'ingresso posso vederlo come $\begin{cases} 0 & \text{altrove} \\ t & 0 \le t \le 2 \end{cases}$

$Y(s) = \dfrac{10}{s+3} \cdot \dfrac{1}{s^2} =$ tra 0 e 2. che è molto più semplice

$= \dfrac{K_{11}}{s^2} + \dfrac{K_{12}}{s} + \dfrac{K_2}{s+3}$ $\quad K_{11} = \left.\dfrac{10}{s+3}\right|_{s=0} = \dfrac{10}{3}$ $\quad K_2 = \left.\dfrac{10}{s^2}\right|_{s=-3} = \dfrac{10}{9}$

$\rho = 3 - 0 = 3 > 1$ $\quad K_{12} + K_2 = 0$ $\quad K_{12} = -\dfrac{10}{9}$

$y(t) = \dfrac{10}{3}t - \dfrac{10}{9} + \dfrac{10}{9}e^{-3t}$ per $0 \le t < 2$

$t > 2$? il sistema è in evoluzione libera perché non c'è ingresso. L'uscita è data dalla combinazione lineare dei modi del sistema

MODI DI $\Sigma = \{e^{-3t}\}$ perché $s = -3$ unico polo di G

$y(t) = Ce^{-3t}$ per $t > 2$

$y(t) \in C^{\rho-1,\infty} \rightarrow y(t) \in C^{\overset{\rho=1 \text{ in } G}{\downarrow}0,\infty} \rightarrow y(t)$ continua

$y(2) = \underbrace{\dfrac{10}{3}2 - \dfrac{10}{9} + \dfrac{10}{9}e^{-6}}_{y(2^-)} = \underbrace{C \cdot e^{-6}}_{y(2^+)}$ $\quad \dfrac{50}{9} + \dfrac{10}{9}e^{-6} = C \cdot e^{-6}$

$C = \dfrac{50}{9}e^{6} + \dfrac{10}{9}$ $\quad y(t) = \left(\dfrac{50}{9}e^{6} + \dfrac{10}{9}\right)e^{-3t}$ per $t > 2$

3) $L(s) = \frac{10}{s(s+3)}$ $\begin{cases} Y(s) = L(s) \cdot E(s) \\ E(s) = R(s) - Y(s) \end{cases}$ $Y(s) = L(s) R(s) - L(s) Y(s)$

$Y(s)(1 + L(s)) = L(s) R(s)$ $Y(s) = \frac{L(s)}{1+L(s)} R(s)$ $G(s) = \frac{L(s)}{1+L(s)} = \frac{\frac{10}{s(s+3)}}{1+\frac{10}{s(s+3)}} =$

$= \frac{10}{s^2+3s+10}$ $G(0) =$ GUADAGNO STATICO $= 1$

$G(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$ $\omega_n = \sqrt{10}$ $\delta = \frac{3}{2\sqrt{10}} = 0,4743$ $T_s = \frac{1,8}{\omega_n} = 0,57$ sec

$T_a = \frac{3}{\delta\omega_n} = 2$ sec

Sovraelongazione $= 100\, e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}} = 18,4\%$

16/04/09

$u(t) \in C^{p,\infty} \Rightarrow y(t) \in C^{p+\rho,\infty}$

$a_n (D^p y_+ - D^p y_-) = b_n (u_+ - u_-)$

$a_{n-1} (D^p y_+ - D^p y_-) + a_n (D^{p+1} y_+ - D^{p+1} y_-) = b_{n-1} (u_+ - u_-) + b_n (Du_+ - Du_-)$

$\vdots$

$a_1 (D^p y_+ - D^p y_-) + \ldots + a_n (D^{n-1} y_+ - D^{n-1} y_-) = b_1 (u_+ - u_-) + \ldots + b_m (D^{m-1} u_+ - D^{m-1} u_-)$

1) se $u \in C^0 \rightarrow y \in C^\rho$ $\rho = n - m$

2) se $u \in C^1 \rightarrow y \in C^{\rho+1}$

$\vdots$

m) se $u \in C^{m-1} \rightarrow y \in C^{\rho+m-1} \Leftrightarrow y \in C^{n-1}$

Dedurre le condizioni che legano y_+, y_-, u_+, u_- e le sue derivate

$a_3 D^3 y(t) + a_2 D^2 y(t) + a_1 D y(t) + a_0 y(t) = b_2 D^2 u(t) + b_1 D u(t) + b_0 u(t)$

$a_3 \neq 0$ $b_2 \neq 0$

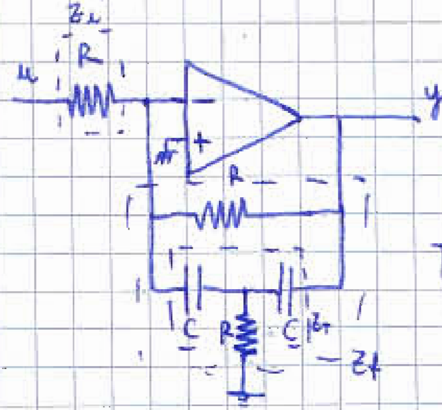
Introduzione [illegible] derivata generalizzata

$$D^{*}u = Du + (u_+ - u_-)\,\delta(t)$$
$$D^{*2}u = D^2u + (Du_+ - Du_-)\,\delta(t) + (u_+ - u_-)\,\delta^{(1)}(t)$$
$$D^{*3}u = D^3u + (D^2u_+ - D^2u_-)\,\delta(t) + (Du_+ - Du_-)\,\delta^{(1)}(t) + (u_+ - u_-)\,\delta^{(2)}(t)$$

$$a_3D^3y + a_3(D^2y_+ - D^2y_-)\delta(t) + a_3(Dy_+ - Dy_-)\delta^{(1)}(t) + a_3(y_+ - y_-)\delta^{(2)}(t) + a_2D^2y +$$
$$+ a_2(Dy_+ - Dy_-)\delta(t) + a_2(y_+ - y_-)\delta^{(1)}(t) + a_1Dy + a_1(y_+ - y_-)\delta(t) + a_0y =$$
$$= b_2D^2u + b_2(Du_+ - Du_-)\delta(t) + b_2(u_+ - u_-)\delta^{(1)}(t) + b_1Du + b_1(u_+ - u_-)\delta(t) + b_0\cdot u$$

Uso il principio di identità delle funzioni impulsive

$$\begin{array}{l} \delta^{(2)} \\ \delta^{(1)} \\ \delta^{(0)} \\ \\ \end{array} \begin{cases} a_3(y_+ - y_-) = 0 \longrightarrow y_+ = y_- \\ a_3(Dy_+ - Dy_-) + \cancel{a_2(y_+ - y_-)} = b_2(u_+ - u_-) \rightarrow Dy_+ = Dy_- + \frac{b_2}{a_3}(u_+ - u_-) \\ a_3(D^2y_+ - D^2y_-) + a_2(Dy_+ - Dy_-) + \cancel{a_1(y_+ - y_-)} = b_2(Du_+ - Du_-) + b_1(u_+ - u_-) = 0 \\ \hookrightarrow D^2y_+ = D^2y_- + \frac{a_3b_1 - a_2b_2}{a_3^2}(u_+ - u_-) + \frac{b_2}{a_3}(Du_+ - Du_-) \end{cases}$$



$T(s) = ?$

MODI = ?

$$T(s) = \frac{Y}{U} = -\frac{Z_f}{Z_i}$$

$$Z_f = \frac{1}{sC} + \frac{1}{sC} + \frac{\frac{1}{sC}\cdot\frac{1}{sC}}{R} = \frac{2}{sC} + \frac{1}{s^2C^2R} =$$
$$= \frac{2RCs+1}{s^2C^2R}$$

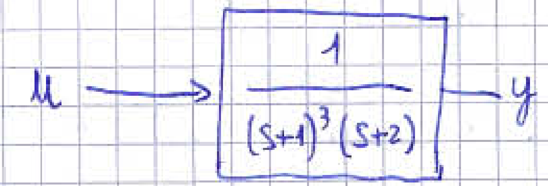
$$Z_f = \frac{\frac{2RCs+1}{s^2C^2}}{R + \frac{2RCs+1}{RC^2s^2}} = \frac{\frac{2RCs+1}{\cancel{s^2C^2}}}{\frac{R^2C^2s^2+2RCs+1}{R\cancel{C^2s^2}}} = \frac{2R^2Cs+R}{(RCs+1)^2}$$

$$T(s) = -\frac{Z_f}{R} = -\frac{2RCs+1}{(RCs+1)^2}$$

POLI $\Rightarrow s = -\frac{1}{RC}$ molteplicità 2.

ZERI $\Rightarrow s = -\frac{1}{2RC}$

$\text{MODI} = \left\{ e^{-\frac{t}{RC}} ; te^{-\frac{t}{RC}} \right\}$ $\qquad$ $\mathcal{L}[t^n e^{\alpha t}] = \frac{n!}{(s-\alpha)^{n+1}}$



$u(t) = 1(t)$

$$Y(s) = G(s)\cdot U(s) = \frac{1}{(s+1)^3(s+2)s} = \frac{A}{s} + \frac{B}{(s+1)^3} + \frac{C}{(s+1)^2} + \frac{D}{s+1} + \frac{E}{s+2}$$

$$A = \frac{1}{(s+1)^3(s+2)}\bigg|_{s=0} = \frac{1}{2} \qquad B = \frac{1}{(s+2)s}\bigg|_{s=-1} = -1 \qquad E = \frac{1}{(s+1)^3\cdot s}\bigg|_{s=-2} = \frac{1}{2}$$

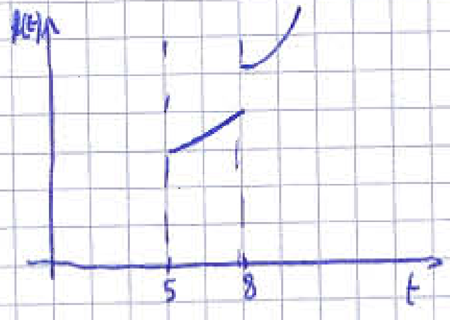
$$C = D\cdot\frac{1}{(s+2)s}\bigg|_{s=-1} = \frac{-2s-2}{(s^2+2s)^2}\bigg|_{s=-1} = 0 \qquad \Sigma R = 0 \text{ perché } \rho = 5 > 1$$

$$A + D + E = 0 \qquad D = -1 \qquad Y(s) = \frac{1/2}{s} + \frac{-1}{(s+1)^3} + \frac{-1}{s+1} + \frac{1/2}{s+2}$$

$$y(t) = \frac{1}{2} - \frac{1}{2}t^2e^{-t} - e^{-t} + \frac{1}{2}e^{-2t} \quad t \geq 0$$

ESERCITAZIONE 5

$\boxed{1}$ $f: \mathbb{R} \to \mathbb{R} \quad t \to f(t) \qquad f(t) = t\cdot 1(t-5) + (t-7)^3\cdot 1(t-8)$



$$f(t) = \begin{cases} 0 & \text{per } t<5 \\ t & \text{per } 5<t<8 \\ t+(t-7)^3 & \text{per } t>8 \end{cases} \qquad Df(t) = \begin{cases} 0 & \text{per } t<5 \\ 1 & \text{per } 5<t<8 \\ 1+3(t-7)^2 & \text{per } t>8 \end{cases}$$

$$Df(t) = 1(t-5) + 3(t-7)^2 1(t-8)$$

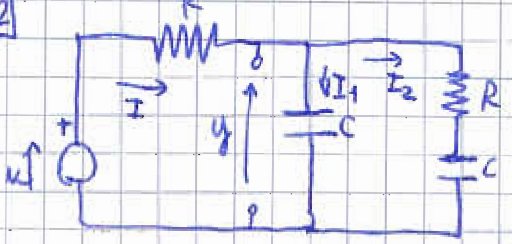
$$D^* f(t) = Df(t) \quad \forall t \in \mathbb{R} - \overset{\text{istanti discont.}}{\{5,8\}} \qquad D^* f(t) = Df(t) + \big(f(5^+) - f(5^-)\big)\delta(t-5) + \big(f(8^+)-f(8^-)\big)\delta($$

$$f(5^-) = 0 \quad f(5^+) = 5 \quad f(8^-) = 8 \quad f(8^+) = 9$$

$$D^* f(t) = 1(t-5) + 3(t-7)^2 \cdot 1(t-8) + 5\,\delta(t-5) + \delta(t-8)$$

$$D^{*2} f(t) = D^2 f(t) + \big(Df(t_x^+) - Df(t_x^-)\big)\delta(t-t_x) + \big(f(t_x^+) - f(t_x^-)\big)\delta^{(1)}(t-t_x)$$

$\boxed{2}$



$$T = RC \qquad I = I_1 + I_2 = \frac{U(s) - Y(s)}{R}$$

$$I_1 = \frac{Y(s)}{\frac{1}{sC}} = sC\,Y(s) \qquad I_2 = \frac{Y(s)}{R + \frac{1}{sC}} = \frac{sC}{1+RsC}\,Y(s)$$

$$U(s) - Y(s) = R \cdot \left(sC + \frac{sC}{1+RsC}\right) Y(s)$$

$$= R\left(\frac{sC + s^2C^2R + sC}{1+RsC}\right) Y(s) = \frac{2RsC + R^2s^2C^2}{1+RsC}\,Y(s)$$

$$U(s) = \frac{1 + RsC + 2RsC + R^2s^2C^2}{1+RsC}\,Y(s) = \frac{R^2C^2s^2 + 3RCs + 1}{1+RCs}\,Y(s)$$

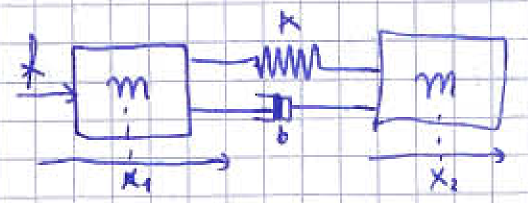
$$Y(s) = \frac{1+Ts}{T^2s^2+3Ts+1}\,U(s) \qquad G(s) = \frac{1+Ts}{T^2s^2+3Ts+1}$$

$$T^2D^2y + 3TDy + y = +u + TDu$$

$$T^2s^2 + 3Ts + 1 = 0 \qquad p_{1,2} = \frac{-3T \pm \sqrt{9T^2-4T^2}}{2T^2} = \frac{-3T \pm \sqrt{5}\,T}{2T^2} = \frac{-3 \pm \sqrt{5}}{2T}$$

$$\text{MODI: } \left\{ e^{\frac{-3+\sqrt{3}}{2T}t} ; e^{\frac{-3-\sqrt{5}}{2T}t} \right\}$$

$\boxed{3}$



$$\begin{cases} mD^2x_1 = f + k(x_2 - x_1) + b(Dx_2 - Dx_1) \\ mD^2x_2 = -k(x_2 - x_1) - b(Dx_2 - Dx_1) \end{cases}$$

$$\begin{cases} (mD^2 + k + bD)x_1 = f + kx_2 + bDx_2 \\ (+k + bD)x_1 = (+mD^2 + k + bD)x_2 \end{cases}$$

$$\begin{cases} (k+bD)(mD^2+k+bD)x_1 = kf + bDf + (k^2 + bkD + bkD + b^2D^2)x_2 \\ (k+bD)(mD^2+k+bD)x_1 = (m^2D^4 + k^2 + b^2D^2 + 2kmD^2 + 2mbD^3 + 2kbD)x_2 \end{cases}$$

$$(\cancel{k^2} \cancel{+2bkD} + \cancel{b^2D^2} - m^2D^4 \cancel{-k^2} - \cancel{b^2D^2} - 2kmD^2 - 2mbD^3 \cancel{-2kbD})x_2 = -kf - bDf$$

$$m^2D^4x_2 + 2mbD^3x_2 + 2kmD^2x_2 = bDf + kf$$

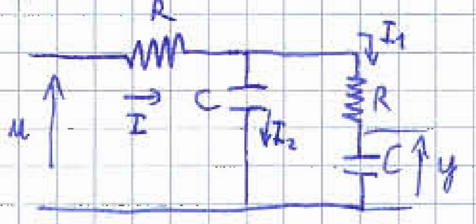
$$T_{x_2,f} = \frac{bs+k}{m^2s^4 + 2mbs^3 + 2kms^2} = \frac{bs+k}{s^2(m^2s^2 + 2mbs + 2km)}$$

$f \in PC^\infty(\mathbb{R})$ due istanti di discontinuità: $t_1 = 0$, $t_2 = 3$

$$D^*f(t) = Df(t) + (f(0^+) - f(0^-))\delta(t) + (f(3^+) - f(3^-))\delta(t-3)$$

$$D^{2*}f(t) = D^2f(t) + (f(0^+) - f(0^-))\delta^{(1)}(t) + (f(3^+) - f(3^-))\delta^{(1)}(t-3) + (Df(0^+) - Df(0^-))\delta(t) +$$
$$+ (Df(3^+) - Df(3^-))\delta(t-3)$$

$$D^{3*}f(t) = D^3f(t) + (f(0^+) - f(0^-))\delta^{(2)}(t) + (f(3^+) - f(3^-))\delta^{(2)}(t-3) + (Df(0^+) - Df(0^-))\delta^{(1)}(t) +$$
$$+ (Df(3^+) - Df(3^-))\delta^{(1)}(t-3) + (D^2f(0^+) - D^2f(0^-))\delta(t) + (D^2f(3^+) - D^2f(3^-))\delta(t-3).$$



$T(s) = ?$

e.d. = ?

$T = RC$

derivatore partitore di corrente

$\downarrow$

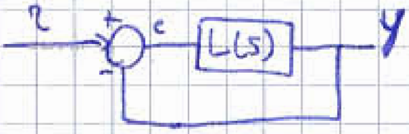
$U = Z \cdot I \qquad Z = R + \frac{RCs+1}{Cs(RCs+2)} \qquad Y = \frac{I_1}{sC} = \frac{1}{sC} \cdot I \cdot \frac{\frac{1}{sC}}{\frac{2}{sC}+R} =$

ramo inutile

somma

$$= \frac{1}{sC} \cdot \frac{U}{Z} \cdot \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \dots \Rightarrow T(s) = \frac{1}{T^2s^2+3Ts+1} = \frac{Y}{U}$$

$$T^2D^2y + 3TDy + y = u$$



$L(s) = \frac{7}{(s+1)(s+9)} \qquad y(t) = ? \qquad r(t) = 5 \cdot 1(t)$

$$T(s) = \frac{L(s)}{1+L(s)} = \frac{7}{s^2+10s+16} = \frac{7}{(s+2)(s+8)} \qquad Y = TR = \frac{7}{(s+2)(s+8)} \cdot \frac{5}{s} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+8}$$

$$K_1 = \frac{35}{16} \qquad K_2 = -\frac{35}{12} \qquad K_3 = \frac{35}{48} \qquad y = \frac{35}{16} - \frac{35}{12}e^{-2t} + \frac{35}{48}e^{-8t} \quad t \geq 0$$

$g_s(t) = \frac{1}{2} + e^{-t} - \frac{3}{2}e^{-2t}$ determinare $G(s)$, $y(t)$, $u(t) = \begin{cases} 0 & t<0 \\ 1+t & t \geq 0 \end{cases}$

$$G_s(s) = \frac{1}{s} \cdot G(s) \qquad G(s) = s \cdot G_s(s) \qquad G_s(s) = \frac{1/2}{s} + \frac{1}{(s+1)} + \frac{-3/2}{(s+2)} =$$

$$= \frac{\frac{1}{2}(s+1)(s+2) + s(s+2) - \frac{3}{2}s(s+1)}{s(s+1)(s+2)} = \frac{\frac{1}{2}\cancel{s^2}+\cancel{s}+\cancel{\frac{s}{2}}+1+\cancel{s^2}+2s-\frac{3}{2}\cancel{s^2}-\cancel{\frac{3}{2}s}}{s(s+1)(s+2)} = \frac{1+2s}{s(s+1)(s+2)}$$

$$G(s) = \frac{1+2s}{(s+1)(s+2)} \qquad Y(s) = G(s) \cdot U(s) \qquad U(s) = \frac{1}{s} + \frac{1}{s^2} = \frac{s+1}{s^2}$$

21/04/09

$$G(s)=\frac{b_2 s^2+b_1 s+b_0}{(s-p_1)(s-p_2)^2\left[(s-\sigma)^2+\omega^2\right]^2}$$

poli : $p_1, p_2, p_2, \sigma \pm j\omega, \sigma \pm j\omega$

$$y_L(t)=c_1 e^{p_1 t}+c_2 e^{p_2 t}+c_3 t e^{p_2 t}+c_4 e^{\sigma t}\operatorname{sen}(\omega t+\varphi_1)+c_5 t e^{\sigma t}\operatorname{sen}(\omega t+\varphi_2)$$

Stabilità asintotica, perché $\lim\limits_{t\to+\infty} t^n e^{-at}=0 \quad \begin{matrix}\forall a>0\\ \forall n\in\mathbb{N}\end{matrix}$

27/04/09

$$G(s)=\frac{10(s+5)}{(s+1)^2(s+10)} \qquad G(j\omega)=\frac{10(j\omega+5)}{(j\omega+1)^2(j\omega+10)}$$

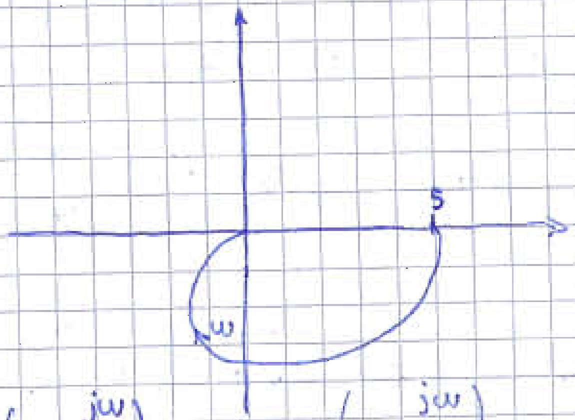
→ NYQUIST

→ BODE

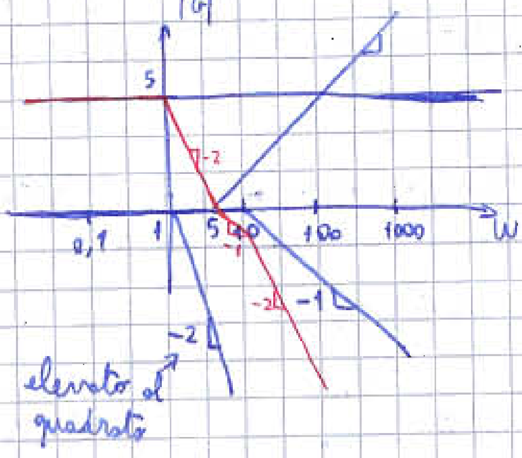
$$\lim_{\omega\to 0^+} G(j\omega)=\frac{50}{10}=5$$

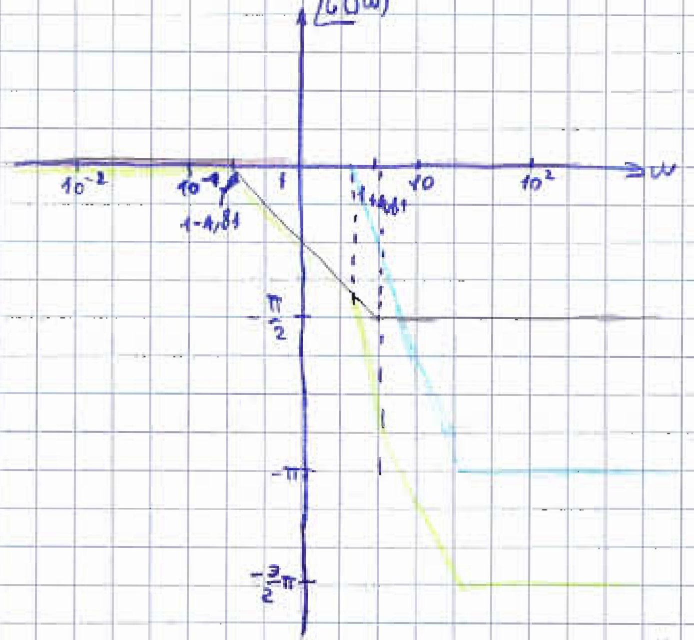
$$\lim_{\omega\to+\infty} G(j\omega)=0 \qquad \arg G(j\omega)=0+\operatorname{arctg}\frac{\omega}{5}-2\operatorname{arctg}\frac{\omega}{1}-\operatorname{arctg}\frac{\omega}{10}:$$

$$\lim_{\omega\to 0^+}\arg G(j\omega)=0 \qquad \lim_{\omega\to+\infty}\arg G(j\omega)=\frac{\pi}{2}-2\frac{\pi}{2}-\frac{\pi}{2}=-\pi$$

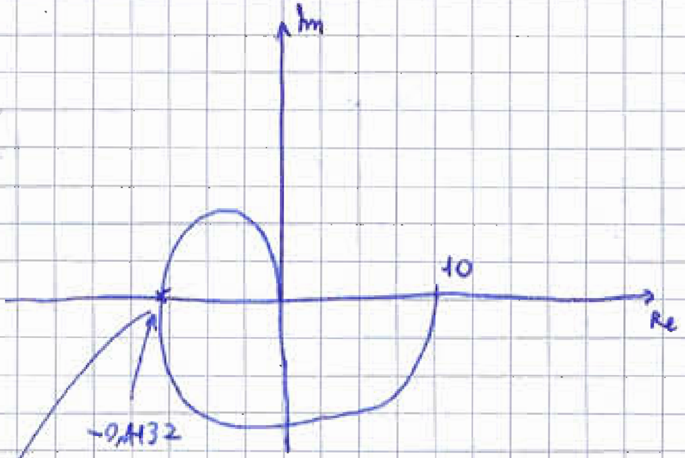


$$G(j\omega)=\frac{5\cancel{0}\left(1+\frac{j\omega}{5}\right)}{(j\omega+1)^2\,\cancel{10}\left(1+\frac{j\omega}{10}\right)}=\frac{5\left(1+\frac{j\omega}{5}\right)}{(j\omega+1)^2\left(1+\frac{j\omega}{10}\right)}$$





BODE



$$|G_1(j\omega)| = \frac{10}{\sqrt{1+\omega^2}\cdot\sqrt{1+\omega^2/10^2}^{\,2}} =$$

$$= \frac{10^3}{\sqrt{1+\omega^2}\cdot\left(10^2+\omega^2\right)}$$

$$\angle G_1(j\omega) = 0 - \text{arctg}\,\frac{\omega}{1} - 2\,\text{arctg}\,\frac{\omega}{10} = -\text{arctg}\,\omega - 2\,\text{arctg}\,\frac{\omega}{10}$$

$-\pi$ se fosse -10

$$\lim_{\omega\to+\infty} \angle G_1(j\omega) = -\frac{\pi}{2} - \pi = -\frac{3}{2}\pi$$

a che ω ha $\arg \angle G_1(j\omega) = -\pi$

$+\text{arctg}\,\omega + 2\,\text{arctg}\,\frac{\omega}{10} = +\pi$ $\quad \text{arctg}\,\frac{\omega}{10} + \text{arctg}\,\frac{\omega}{10} = \pi - \text{arctg}\,\omega$ applico tg

$$tg(\alpha+\beta) = \frac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha\, tg\,\beta} \qquad \frac{\frac{\omega}{10}+\frac{\omega}{10}}{1-\frac{\omega^2}{100}} = \frac{0-\omega}{1-0\cdot\omega}$$

$$-\omega = \frac{\frac{2\omega}{10}}{\frac{100-\omega^2}{100}} \Rightarrow -\omega = \frac{20\omega}{100-\omega^2} \qquad -(-\omega^3+100\omega) = 20\omega$$

$$\omega^3 - 120\omega = 0 \qquad \omega(\omega^2-120) = 0$$

$\omega = \sqrt{120}$ rad/s

$$\left|G_1(j\sqrt{120})\right| = \frac{10}{\sqrt{121}\left(1+\frac{120}{10^2}\right)} \simeq 0{,}4132$$

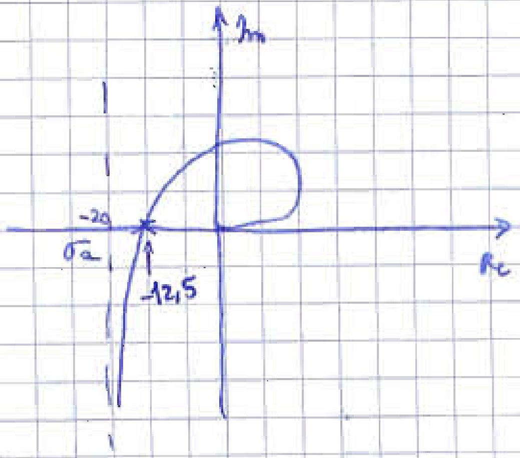
$$G_2(s) = \frac{40(1-s)}{s(s+2)^2}$$

Σ è strettamente proprio (il grafico arriva allo 0), non a fase minima (zero positivo), tipo 1 (polo nell'origine, partirà da $-\infty$)

$$G_2(j\omega) = \frac{40(1-j\omega)}{j\omega(2+j\omega)^2} = \frac{10(1-j\omega)}{j\omega\left(1+\frac{j\omega}{2}\right)^2}$$

$$\arg G_2(j\omega) = -\text{arctg}\,\omega - \frac{\pi}{2} - 2\,\text{arctg}\,\frac{\omega}{2}$$

$$\lim_{\omega \to +\infty} \arg G_2(j\omega) = -\frac{\pi}{2} - \frac{\pi}{2} - 2\frac{\pi}{2} = -2\pi$$



$$G(j\omega) = K\frac{(1+j\omega\tau_1')(1+j\omega\tau_2')\cdots\left(1+\frac{2\delta_1' j\omega}{\omega_n'}+\frac{(j\omega)^2}{\omega_{n}'^2}\right)}{(1+j\omega\tau_1)(1+j\omega\tau_2)\cdots\left(1+\frac{2\delta_i j\omega}{\omega_n}+\frac{(j\omega)^2}{\omega_n}\right)}$$

$$\sigma_0 = K\left(\sum_i \tau_i' + \sum_i \frac{2\delta_i'}{\omega_n} - \sum_i \tau_i - \sum 2\frac{\delta_i}{\omega_n}\right)$$

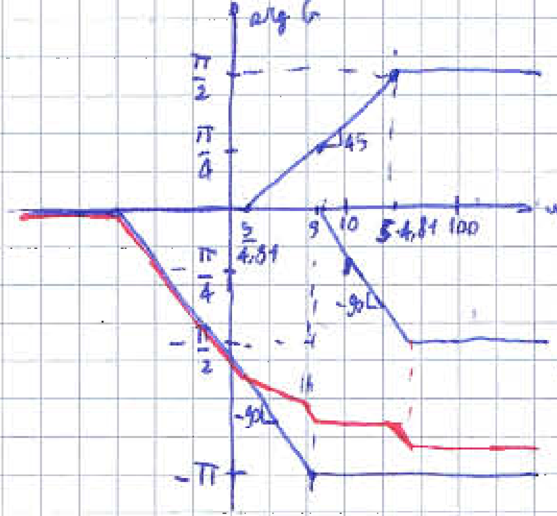
$\tau_1' = -1$ $\qquad$ $\tau_1 = \left(\frac{1}{2}+\frac{1}{2}\right) = 1$ $\qquad$ $\sigma_a = 10(-1-1) = -20$

$+\text{arctg}\,\omega + \frac{\pi}{2} + 2\,\text{arctg}\,\frac{\omega}{2} = \pi$ $\qquad$ $\text{arctg}\,\omega + 2\,\text{arctg}\,\frac{\omega}{2} = \frac{\pi}{2}$

applico cotg perché $tg\,\frac{\pi}{2} = +\infty$ $\qquad$ $\text{cotg}(\alpha+\beta) = \frac{1-tg\alpha\,tg\beta}{tg\alpha+tg\beta}$

$$\frac{1-\omega\,tg\left[2\,\text{arctg}\,\frac{\omega}{2}\right]}{\cancel{\omega+tg\left[2\,\text{arctg}\,\frac{\omega}{2}\right]}} = 0 \qquad 1-\omega\cdot\frac{\frac{\omega}{2}+\frac{\omega}{2}}{1-\frac{\omega^2}{4}} = 0$$

$$1-\frac{\omega^2}{4}-\omega(\omega) = 0 \qquad 1-\frac{5}{4}\omega^2 = 0 \qquad \omega^2 = \frac{4}{5} \qquad \omega = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

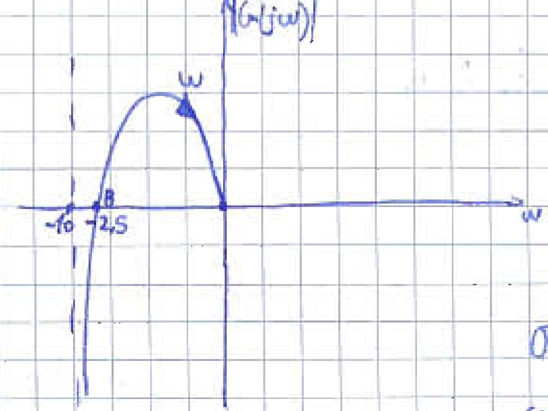


$G(s) = \dfrac{40}{s(s+2)^2}$ sistema di ordine 1 perchè c'è un polo nell'origine.

$\Rightarrow$ asintoto verticale

$G(j\omega) = \dfrac{40}{j\omega(j\omega+2)^2}$ $\qquad \lim_{\omega \to 0} G(j\omega) = \infty \qquad \lim_{\omega \to +\infty} G(j\omega) = 0$

$\arg G(j\omega) = 0 - \dfrac{\pi}{2} - 2\,\mathrm{arctg}\,\dfrac{\omega}{2} \qquad \lim_{\omega \to 0} \arg G(j\omega) = -\dfrac{\pi}{2} \qquad \lim_{\omega \to +\infty} \arg G(j\omega) = -\dfrac{\pi}{2} - 2 \cdot \dfrac{\pi}{2} = -\dfrac{3}{2}$



$G(j\omega) = 40 \cdot \dfrac{1}{4j\omega\left(1+\frac{j\omega}{2}\right)^2} = 10 \cdot \dfrac{1}{j\omega\left(1+\frac{j\omega}{2}\right)^2}$

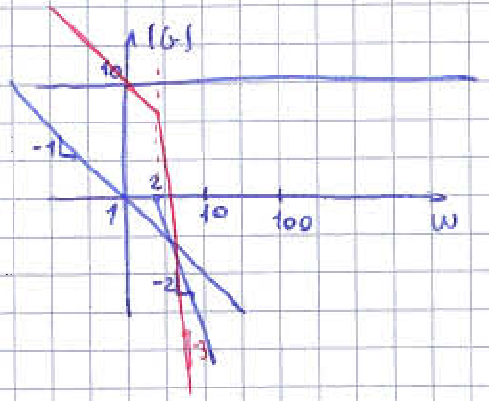
$\sigma_a = \underset{\substack{\uparrow \\ \text{coefficienti} \\ \text{numeratore}}}{K} \left[\sum_{\tau\ \text{sopra}} \tau' - \sum \tau \cdot \cancel{1}\right] = 10\left[-\dfrac{1}{2} - \dfrac{1}{2}\right] = -10$

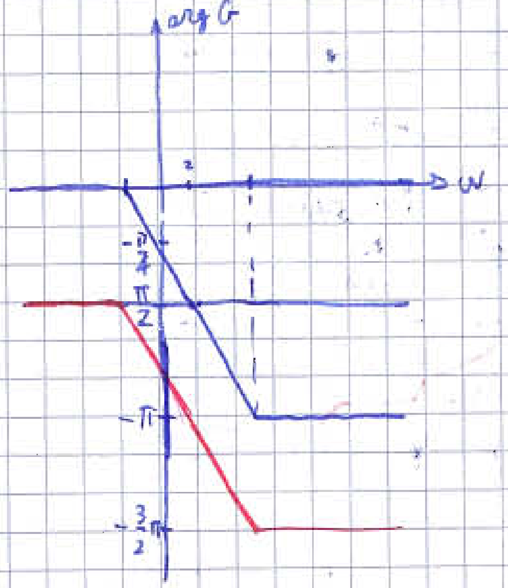
Trovo ω ki B $\arg G = -\pi \Rightarrow -\dfrac{\pi}{2} - 2\,\mathrm{arctg}\,\dfrac{\omega}{2} = -\pi$

$\mathrm{arctg}\,\dfrac{\omega}{2} = \dfrac{\pi}{4} \qquad \dfrac{\omega}{2} = \mathrm{tg}\,\dfrac{\pi}{4} = 1 \qquad \omega = 2$

$|G(j\omega)| = \dfrac{40}{\omega(\omega^2+4)} \qquad |G(2)| = \dfrac{40}{2(8)} = \dfrac{40}{16} = \dfrac{5}{2} = 2{,}5 = B$

$G(j\omega) = \dfrac{40}{4j\omega\left(1+\frac{j\omega}{2}\right)^2} = \dfrac{10}{j\omega\left(1+\frac{j\omega}{2}\right)^2}$





$$G(s) = \frac{k}{s\left[m^2s^3 + 2mbs^2 + \left(b^2 + 2mk\right)s + 2bk\right]} \qquad m, k, b > 0$$

molt. 1 OK ↗ ← controllo con tabella

Dimostrare che il sistema è semplicemente stabile → criterio di Rough

3	m^2	b^2+2mk	0
2	$2mb^2$	$2bk$	0
1	$\boxed{*}$	0	
0	k	0	

$$\boxed{*} \quad \frac{2mb^2\cdot(b^2+2mk) - m^2 2k\cancel{b}}{2mb^2} = b^2m + m^2k$$

$m, b, k > 0$

Tutti positivi → 3 permanenze → semplicemente stabile.

30/04/09

ESERCITAZIONE 6

①

$$G(s) = \frac{b(s)}{a(s)} = \frac{7s^4 + 4s^3 + 5s^2 + 8s + 12}{s^6 + 3s^5 + 10s^4 + 24s^3 + 32s^2 + 48s + 32}$$

6	1	10	32	32	0
5	~~3~~ 1	~~24~~ 8	~~48~~ 16	0	0
4	~~2~~ 1	~~16~~ 8	~~32~~ 16	0	0
3'	0	0	0	0	0
3	~~4~~ 1	~~16~~ 4	0	0	
2	~~4~~ 1	~~16~~ 4	0	0	
1'	0	0	0	0	
1	2	0	0		

RADICI A PARTE REALE NON POSITIVA

$$\gamma_{4,1} = \frac{-\begin{vmatrix} 1 & 10 \\ 3 & 24 \end{vmatrix}}{3} = 2$$

POLINOMIO AUSILIARIO ↓

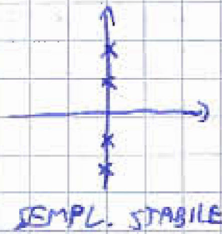
$Q_2(s) = s^4 + 8s^2 + 16$ derivo

$Q_2'(s) = 4s^3 + 16s$

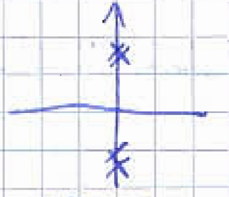
$a_3(s) = s^2+4 \qquad Da_3(s) = 2s$

$a_2(s) = s^4 \pm 8s^2 + 16$ ha grado 4 ⇒ 4 radici
pari ⇒ simmetriche

Posso avere due casi:


SEMPL. STABILE

o


INSTABILE

$\exists\, a_4(s) : a_2(s) = a_4(s) \cdot a_3(s)$

noto che $a_4 = a_3$ perché

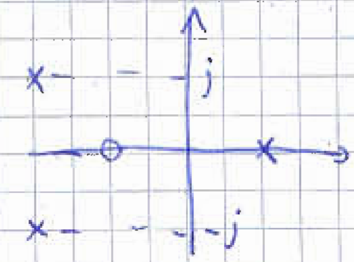
$a_2(s) = [a_3(s)]^2 = (s^2+4)^2 \Rightarrow$ due radici immaginarie molt. 2

$\Downarrow$

SISTEMA INSTABILE

②

$$P(s) = \frac{s+1}{(s-1)(s^2+4s+5)} \qquad K \in \mathbb{R} \qquad s^2+4s+5=0 \Rightarrow s = \frac{-2 \pm \sqrt{4-5}}{1} = -2 \pm j$$



$$T_{ry}(s) = \frac{KP(s)}{1+KP(s)} = \frac{K\frac{s+1}{(s-1)(s^2+4s+5)}}{1+K\frac{s+1}{(s-1)(s^2+4s+5)}} = \frac{K(s+1)}{(s-1)(s^2+4s+5)+K(s+1)} = a(s)$$

$$a(s) = s^3+4s^2+5s-s^2-4s-5+Ks+K = s^3+3s^2+(K+1)s+(K-5)$$

$$\begin{array}{c|ccccc} 3 & 1 & K+1 & 0 & 0 \\ 2 & 3 & K-5 & 0 & 0 \\ 1 & 2K+8 & 0 & 0 & 0 \\ 0 & -\frac{(2K+8)(K-5)}{(2K+8)} & 0 & 0 & \end{array}$$

1

$5-K$

$$-\frac{K-5-3(K+1)}{3} = \frac{2K+8}{3}$$ moltiplico per 3

$$\begin{cases} 2K+8>0 \rightarrow K>-4 \\ K-5>0 \rightarrow \boxed{K>5} \end{cases} \longrightarrow \Sigma \text{ ASINTOTICAMENTE STABILE}$$

Per $K=5$ l'ultima riga si annulla e la penultima diventa 48

$$a(s) = s^3+3s^2+(K+1)s = s^3+3s^2+6s = \underset{\substack{\uparrow \\ \text{POLO} \\ \text{ORIGINE.}}}{s}\underbrace{(s^2+3s+6)}_{\text{POLO IMMAGINARIO COMPLESSE CONIUGATE}} = 0 \Rightarrow \Sigma \text{ SEMPLICEMENTE STABILE}$$

$-4<K<5$

$$\begin{array}{c|cc} 3 & 1 & \\ 2 & 3 & P \\ 1 & >0 & P \\ 0 & <0 & V \end{array}$$

$n_-(a)=2$ radici a parte reale negativa

$n_+(a)=1$ " " " " positiva

INSTABILE

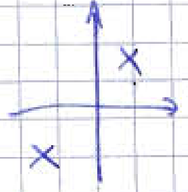
$K=-4$

$$\begin{array}{c|cc} 3 & 1 & -3 \\ 2 & \cancel{8}\,1 & \cancel{-5}\,-3 \\ 1' & 0 & 0 \\ 1 & 2 & 0 \\ 0 & -3 & 0 \end{array}$$

$a_2(s) = s^2-3 \quad Da_2(s) = 2s$

$\downarrow$

radici simmetriche $\rightarrow$ complesse



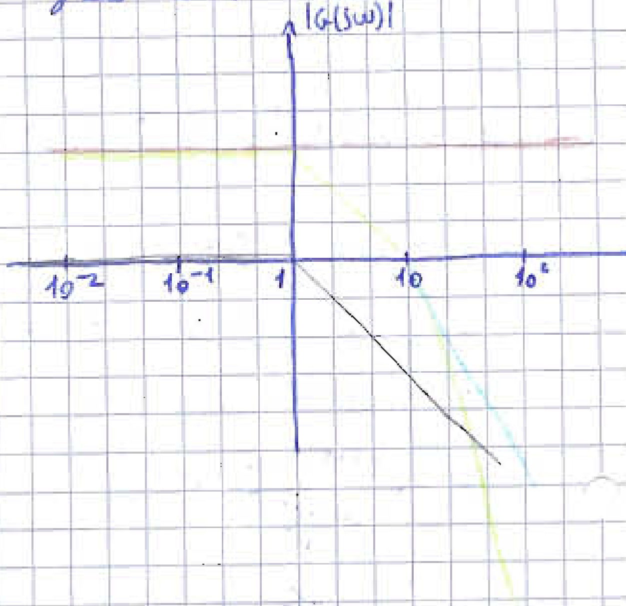
ma dato che ho una variazione

INSTABILE

③ $$G_1(s) = \frac{10^3}{(s+1)(s+10)^2}$$

sistema di tipo 0 perchè non ha poli o zeri nell'origine / strettamente proprio perchè grado num < grado den. , fase minima

$$G_1(j\omega) = \frac{10^3}{(1+j\omega)\left(1+\frac{j\omega}{10}\right)^2 \cdot 10^2} = \frac{10}{(1+j\omega)\left(1+\frac{j\omega}{10}\right)^2}$$



$$|G_2(j\omega)| = \frac{10\sqrt{1+\omega^2}}{|j\omega|\left(1+\frac{\omega^2}{4}\right)} \Rightarrow \left|G_2\left(j\sqrt{\tfrac{4}{5}}\right)\right| = \frac{10\sqrt{1+\frac{4}{5}}}{\sqrt{\frac{4}{5}}\left(1+\frac{1}{5}\right)} = 12,5$$

11/5/09

$$P(s) = \frac{10}{(s+1)(s+2)(s+10)} \qquad L(s) = \frac{380}{(s+1)(s+2)(s+10)}$$

$C(s) = 38$

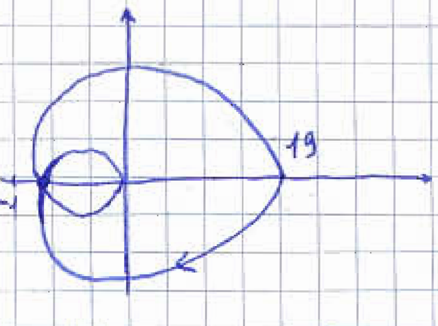
$r(t) = 60 \cdot 1(t)$

$$L(j\omega) = \frac{380}{(j\omega+1)(j\omega+2)(j\omega+10)} =$$

$$|L| = \frac{380}{\sqrt{1+\omega^2}\,\sqrt{4+\omega^2}\,\sqrt{100+\omega^2}} \qquad \arg L = 0 - \text{arctg}\,\omega - \text{arctg}\,\frac{\omega}{2} - \text{arctg}\,\frac{\omega}{10}$$

$$\lim_{\omega\to 0^+} |L| = 19 \qquad \lim_{\omega\to +\infty} |L| = 0$$

$$\lim_{\omega\to 0^+} \arg L = 0 \qquad \lim_{\omega\to +\infty} \arg L = -\frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} = -\frac{3}{2}\pi$$



L(jω) non ha poli a parte reale positiva, quindi non gira neanche una volta attorno a −1.

$\arg L(j\omega) = -\pi$ $\quad \text{arctg}\,\omega + \text{arctg}\,\frac{\omega}{2} + \text{arctg}\,\frac{\omega}{10} = \pi$ Trovo $\omega_p \rightarrow |L(\omega_p)| \cdots$

oppure

$$1+\alpha L(s) = 0 \Rightarrow 1+\alpha \cdot \frac{380}{(s+1)(s+2)(s+10)} = 0 \qquad s^3+13s^2+32s+20+380\alpha = 0$$

3	1	32	0
2	13	$20+380\alpha$	0
1	$13\cdot 32-$ $20-38\alpha$	0	0
0	$20+380\alpha$		

serve una riga tutta nulla

$\Downarrow$

pongo $13\cdot 32-20-38\alpha=0 \Rightarrow \alpha = 1{,}04$ MARGINE AMPIEZZA

l'intersezione è $-\frac{1}{\alpha} = -0{,}9516$

$e_r = r(\infty) - c(\infty)$ errore a regime ? $e_r = \frac{R_0}{1+K_p}$ con $K_p = \lim_{s\to 0} L(s) = 19$

$$e_r = \frac{60}{1+19} = 3$$

① Determinare $y(t)$ per $t>0$ del sistema $G(s) = \frac{8}{s+2}$ all'ingresso $u(t) = 4\,\text{sen}(2t)\cdot 1(t) = U\,\text{sen}(\omega t)$

$$Y(s) = G(s)\cdot U(s) = \frac{8}{s+2}\cdot 4 \cdot \frac{\overset{\omega}{2}}{s^2+\underset{\omega^2}{4}} = \frac{64}{(s+2)(s^2+4)} = \frac{64}{(s+2)(s+2j)(s-2j)} =$$

$$= \frac{A}{s+2} + \frac{B}{s+2j} + \frac{\bar{B}}{s-2j}$$

$\downarrow$

$Ae^{-2t} + \dots\dots$

dal teorema di analisi armonica $Y(\omega)\cdot \text{sen}(\omega t + \varphi(\omega))$

$$F(j\omega) = \frac{Y(\omega)}{U} e^{j\varphi(\omega)}$$

$$\Rightarrow 8e^{-2t} + 8\sqrt{2}\,\text{sen}\left(2t - \frac{\pi}{4}\right)$$

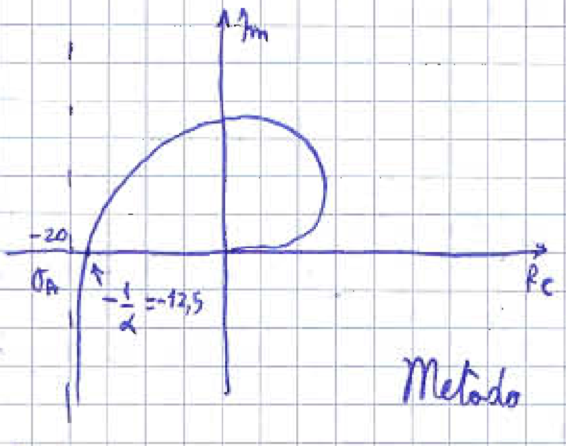
ESERCITAZIONE 7

$$L_2(s) = \frac{40(1-s)}{s(s+2)^2} \qquad L(j\omega) = \frac{\overset{10}{\cancel{40}}(1-j\omega)}{\cancel{4}j\omega\left(1+\frac{j\omega}{2}\right)^2} \qquad |L(j\omega)| = \frac{10\cdot\sqrt{1+\omega^2}}{|\omega|\left(1+\frac{\omega^2}{4}\right)} = \frac{40\sqrt{1+\omega^2}}{|\omega|(4+\omega^2)}$$

$$\arg L(j\omega) = -\text{arctg}\,\omega - \frac{\pi}{2} - 2\,\text{arctg}\frac{\omega}{2} \qquad \lim_{\omega\to 0^+} |L(j\omega)| = \infty$$

$$\lim_{\omega\to 0^+} \arg L(j\omega) = -\frac{\pi}{2}$$

$$\lim_{\omega \to +\infty} |L(j\omega)| = 0 \qquad \lim_{\omega \to +\infty} \arg L(j\omega) = -2\pi$$



$$\sigma_A = K\left(\sum_i \tau_i' + \sum_i \frac{2\delta_i'}{\omega_{n_i}'} - \sum_i \tau_i - \sum \frac{2\delta_i}{\omega_{n i}}\right) =$$

$$= 10\left(-1 - 2 \cdot \frac{1}{2}\right) = -20$$

Metodo Routh

$\exists\, \alpha \in \mathbb{R}_+ : 1 + \alpha L(s) = 0$ ha radici puramente immaginarie

$\exists\, \omega \in \mathbb{R}_+ : L(\pm j\omega) = -\frac{1}{\alpha}$

$$1 + \frac{40(1-s)}{s(s+2)^2} \cdot \alpha = 0 \qquad s(s+2)^2 + \overset{\alpha'}{\overbrace{40\alpha}}(1-s) = 0$$

$$s^3 + 4s^2 + 4s + \alpha' - \alpha' s = 0 \qquad s^3 + 4s^2 + (4 - \alpha')s + \alpha' = 0$$

3	1	$4-\alpha'$	0
2	4	α'	0
1	$f(\alpha')$	0	← deve essere tutti zeri
0			

$$f(\alpha') = -\frac{1}{4}\left(\alpha' - 4(4-\alpha')\right) = 0 \qquad \alpha' - 16 + 4\alpha' = 0 \qquad \alpha' = \frac{16}{5} = 40\alpha$$

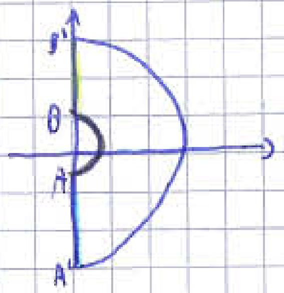
$$\alpha = \frac{16}{200} = \frac{4}{50} = \frac{2}{25} \qquad -\frac{1}{\alpha} = -\frac{25}{2} = -12,5$$

$$4s^2 + \frac{16}{5} = 0 \Rightarrow s_{1,2} = \pm j\sqrt{\frac{4}{5}} \rightarrow \omega_p = \sqrt{\frac{4}{5}}$$ frequenza di taglio

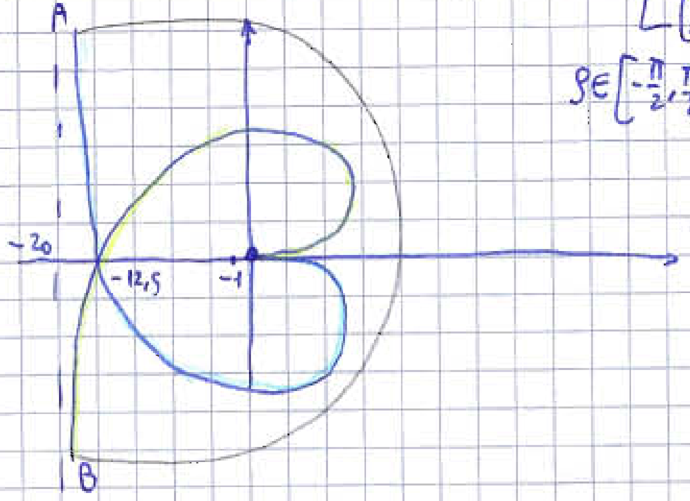
↑ polinomio ausiliario 2

Costruisco ora il diagramma completo di Nyquist

Contorno di Nyquist $1+L(s)=0$



AB sotto il polo nell'origine



$$L(\rho e^{j\varphi}) = \frac{40(1-\rho e^{j\varphi})}{\rho e^{j\varphi}(\rho e^{j\varphi}+2)^2} \quad \varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

: per $\rho \to 0$ dato che la cfr ha raggio $\to 0$

$$= \frac{\overset{10}{\cancel{40}}}{\cancel{4} \cdot 0} = +\infty$$

$$= \boxed{\frac{10}{\rho}} e^{-j\varphi} \longrightarrow +\infty \text{ raggio}$$

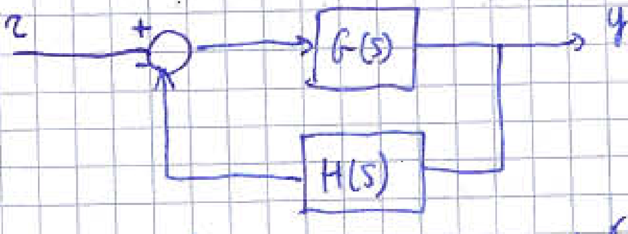
Mentre per la cfr grande A'B' $L(\rho e^{j\varphi})$ con $\rho \to +\infty$, $L(\rho e^{j\varphi}) = 0$ quindi la cfr esterna viene mappata nell'origine.

$\varphi = n_z - n_p = n_z = 2$ INSTABILE

n° giri intorno -1

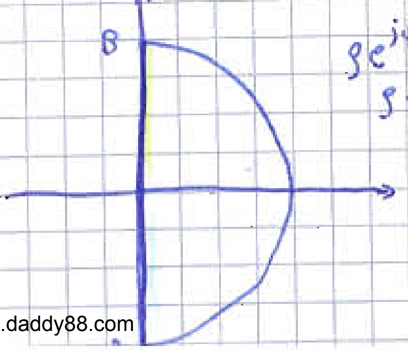
n_z: poli a parte reale positiva

n_p: poli a parte reale negativa



$$T_{ry} = \frac{G(s)}{1+L(s)} \Rightarrow \quad G(s) = \frac{10^3}{(s+1)(s+10)} \quad H(s) = \frac{1}{s+10} \quad L(s) = G(s)\,H(s)$$

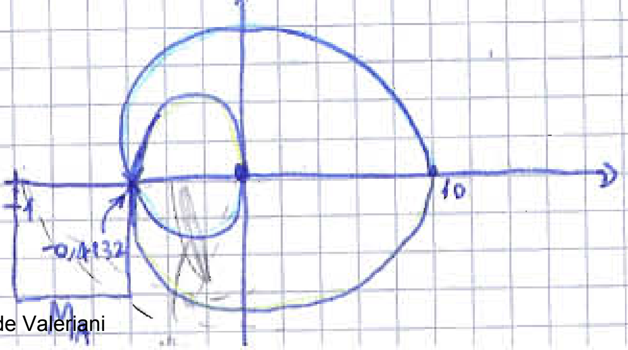
$$L(s) = \frac{10^3}{(s+1)(s+10)^2} \qquad L(j\omega) = \frac{10^3}{100(j\omega+1)\left(1+\frac{j\omega}{10}\right)^2} = \frac{10}{(1+j\omega)\left(1+\frac{j\omega}{10}\right)^2}$$



$\rho e^{j\varphi}$ con $\rho \to +\infty$

$\Downarrow$

0



$$|L(j\omega)| = \frac{10^3}{\sqrt{1+\omega^2}\,(10^2+\omega^2)} \qquad \arg L(j\omega) = -\text{arctg}\,\omega - 2\,\text{arctg}\,\frac{\omega}{10}$$

$$\omega \to 0^+ \begin{cases} |L(j\omega)| = 10 \\ \arg L(j\omega) = 0 \end{cases} \qquad \omega \to +\infty \begin{cases} |L(j\omega)| = 0 \\ \arg L(j\omega) = -\frac{3}{2}\pi \end{cases}$$

$\arg L(j\omega_p) = -\pi$ per trovare x

$$\text{arctg}\,\omega_p + 2\,\text{arctg}\,\frac{\omega_p}{10} = \pi \qquad \text{arctg}\,\frac{\omega_p}{10} + \text{arctg}\,\frac{\omega_p}{10} = \pi - \text{arctg}\,\omega_p \quad \text{applico tg}$$

$$\frac{\frac{\omega_p}{10} + \frac{\omega_p}{10}}{1 - \frac{\omega_p^2}{100}} = -\omega_p \qquad \frac{\frac{\cancel{2}\omega_p}{\cancel{10}}}{\frac{100-\omega_p^2}{5\cancel{100}}} = -\cancel{\omega_p} \qquad \frac{20}{100-\omega_p^2} = 1$$

$$\omega_p^2 - 100 = 20 \qquad \omega_p^2 = 120 \qquad \omega_p = \sqrt{120}\ \frac{rad}{s}$$

$$|L(j\omega_p)| = \frac{10^3}{\sqrt{121}\,(10^2+120)} = 0,4132$$

Il sistema è asintoticamente stabile perché il diagramma completo non avvolge -1.

$$M_A \triangleq \frac{1}{|L(j\omega_p)|} = \frac{1}{0,4132} = 2,4 \qquad M_F \triangleq \pi - |\varphi_c| \quad \text{dove } \varphi_c = \arg L(j\omega_c)$$

dove $\omega_c : |L(j\omega_c)| = 1$

$$\frac{10^3}{\sqrt{1+\omega_c^2}\,(10^2+\omega_c^2)} = 1 \qquad 10^3 = \sqrt{1+\underset{x}{\underset{\downarrow}{\omega_c^2}}}\,(10^2+\omega_c^2) \qquad 10^6 = \underbrace{(1+x)\cdot(10^2+x)^2}_{f(x)}$$

tentativi : $f(50) = 1\,147\,500 > 10^6$

$f(45) = 967\,150 < 10^6 \qquad f(45,95) = 1000100 \simeq 10^6$

$$\omega_c^2 = 45,95 \qquad \omega_c = \sqrt{45,95} \simeq 6,78\ \frac{rad}{s} \qquad \varphi_c = \arg L(j\omega_c) = -\text{arctg}\,\omega_c - 2\,\text{arctg}\,\frac{\omega_c}{10}$$

$$= -1,4243 - 2\cdot 0,5958 = -2,6159\ rad \Rightarrow M_F = \pi - 2,6159 = 0,5257\ rad =$$

$$= 30,12^\circ$$

$$L_3(s) = 10\,\frac{s+1}{s^2(s+5)} \qquad L(j\omega) = \cancel{10}^{\,2}\,\frac{j\omega+1}{\cancel{5}(j\omega)^2(\cancel{5}+\frac{j\omega}{5})} = 2\,\frac{(1+j\omega)}{(j\omega)^2(1+\frac{j\omega}{5})}$$

$$|L(j\omega)| = 2\cdot\frac{\sqrt{1+\omega^2}}{\omega^2\sqrt{1+\frac{\omega^2}{25}}} = 10\,\frac{\sqrt{1+\omega^2}}{\omega^2\sqrt{25+\omega^2}}$$

$$\arg L(j\omega) = \text{arctg}\,\omega - \cancel{2}\cdot\frac{\pi}{\cancel{2}} - \text{arctg}\,\frac{\omega}{5}$$

$\omega \to 0^+$ $\begin{cases} |L(j\omega)| = +\infty \\ \arg L(j\omega) = -\pi \end{cases}$ $\omega \to +\infty$ $\begin{cases} |L(j\omega)| = 0 \\ \text{arctg}\, L(j\omega) = -\pi \end{cases}$

Contorno Nyquist

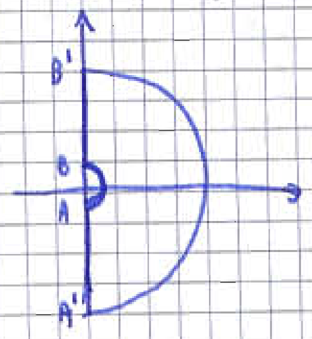
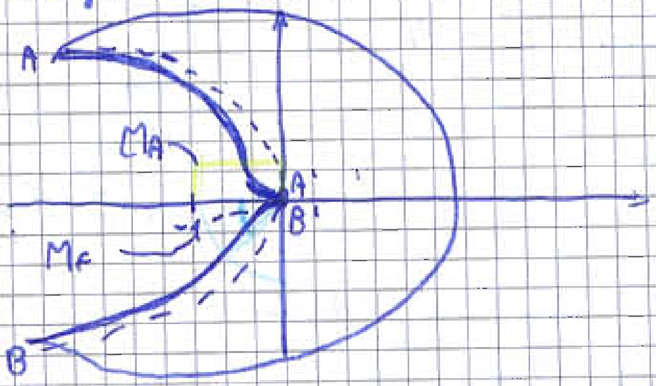


Diagramma polare



$\rho e^{j\varphi}$ $\varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$L(\rho e^{j\varphi}) = 10\cdot\frac{(1+\rho e^{j\varphi})}{(\rho e^{j\varphi})^2(5+\rho e^{j\varphi})} \qquad \rho \to 0 \Rightarrow 2\cdot\underbrace{\frac{1}{\rho^2}}_{\to +\infty} e^{-j2\varphi}$$

In B $\varphi = \frac{\pi}{2} \to -2\varphi = -\pi$

In A $\varphi = -\frac{\pi}{2} \to -2\varphi = \pi$

$\Rightarrow$ all'infinito diventa una circonferenza

Σ ASINTOTICAMENTE STABILE

$$M_A \triangleq \frac{1}{|L(j\omega_\pi)|} = \infty \qquad M_F = \pi - |\varphi_c| \qquad \varphi_c = \arg L(j\omega_c)$$

$$|L(j\omega_c)| = \frac{10\sqrt{1+\omega_c^2}}{\omega_c^2\sqrt{25+\omega_c^2}} = 1 \qquad \omega_c^2 = x \qquad 10\sqrt{1+x} = x\sqrt{25+x}$$

$$100(1+x) = x^2(25+x)$$

$x^3 + 25x^2 - 100x - 100 = 0$? $x=1 \to 200 \neq 26$ $x = 4{,}2 \to 520 \simeq 515$

$x = 4 \to 500 \neq 464$ $\omega_c^2 = 4{,}2 \Rightarrow \omega_c = 2{,}05\ \frac{\text{rad}}{s}$

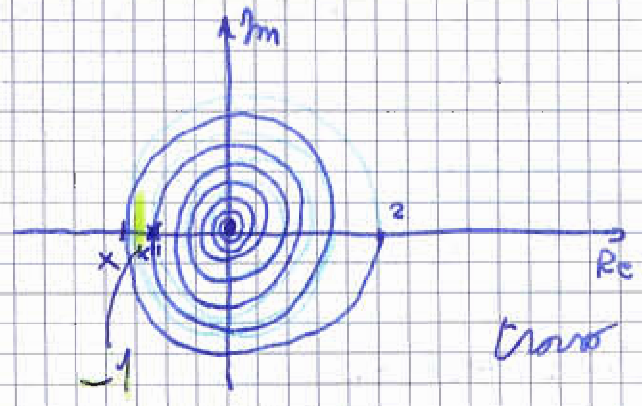
$\varphi_c = \text{arctg}\, \omega_c - \pi - \text{arctg}\, \frac{\omega_c}{5} \simeq -2,1137 \quad M_F = 0,7279 = 41,7°$

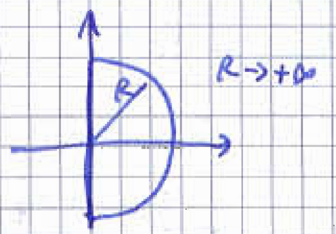
$L_4(s) = 2 \cdot \frac{1-s}{(1+s)^2} e^{-s} \qquad L(j\omega) = 2\, \frac{1-j\omega}{(1+j\omega)^2} e^{-j\omega} \hookrightarrow |\alpha| e^{j\beta} \Rightarrow \begin{matrix} |\alpha| = 1 \\ \beta = -\omega \end{matrix} -$

$\overset{\uparrow}{\text{fase}}$

$|L(j\omega)| = \frac{2\sqrt{1+\omega^2}}{1+\omega^2} = \frac{2}{\sqrt{1+\omega^2}} \qquad \arg L(j\omega) = -\text{arctg}\, \omega - 2\text{arctg}\, \omega - \omega = -3\text{arctg}\, \omega - \omega$

$\omega \to 0 \quad \begin{cases} |L(j\omega)| = 2 \\ \arg L(j\omega) = 0 \end{cases} \qquad \omega \to +\infty \quad \begin{cases} |L(j\omega)| = 0 \\ \arg L(j\omega) \Rightarrow +\infty \end{cases}$





trovo X

$3\,\text{arctg}\, \omega_p + \omega_p = \pi \qquad$ tentativi $\qquad \omega_p = 1 \quad 3,3562 > 3,1416$

$\omega_p = 0,917 \quad 3,1434 \simeq 3,1416$

$|L(j\omega_p)| = \frac{2}{\sqrt{1+0,917^2}} = 1,474 \qquad x = -1,474 = L(j\omega_p)$

trovo x'

$3\,\text{arctg}\, \omega_{3p} + \omega_{3p} = 3\pi \qquad \omega_{3p} = 5,28 \ \frac{rad}{s} \qquad |L(j\omega_{3p})| \simeq 0,3722$

$\psi = n_z - n_p \qquad n_p = 0 \qquad \psi = n_z = 2 \Rightarrow \Sigma$ INSTABILE

19/05/09

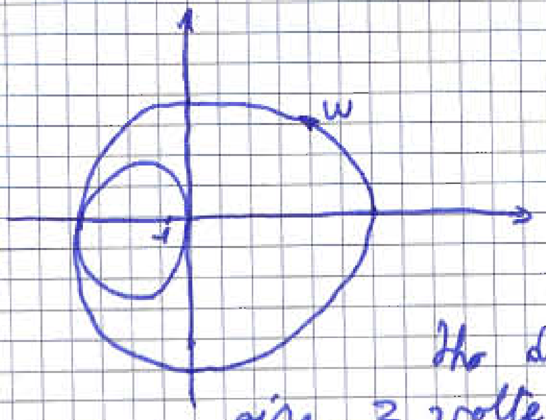
$L(s) = 100 \cdot \frac{s+2}{(s-1)(s-2)} \qquad L(j\omega) = 100\, \frac{\cancel{2}\left(1+\frac{j\omega}{2}\right)}{(1-j\omega)\cancel{2}\left(1-\frac{j\omega}{2}\right)}$

$|L(j\omega)| = 100\, \frac{\cancel{\sqrt{1+\frac{\omega^2}{4}}}}{\sqrt{1+\omega^2}\cancel{\sqrt{1+\frac{\omega^2}{4}}}} = \frac{100}{\sqrt{1+\omega^2}}$

$$\arg L(j\omega) = \operatorname{arctg}\frac{\omega}{2} + \operatorname{arctg}\omega + \operatorname{arctg}\frac{\omega}{2}$$

$\omega \to 0^+$ $\begin{cases} |L(j\omega)| = 100 \\ \arg L(j\omega) = 0 \end{cases}$ $\omega \to +\infty$ $\begin{cases} |L(j\omega)| = 0 \\ \arg L(j\omega) = \frac{3}{2}\pi \end{cases}$

$+ \Rightarrow$ verso antiorario



$\arg L(j\omega_\pi) = \pi$ ...

$|L(j\omega_\pi)| = -33,\bar{3}$

Ho due poli a parte reale positiva. Il vettore gira 2 volte intorno a -1.

$\Rightarrow$ sistema asintoticamente stabile

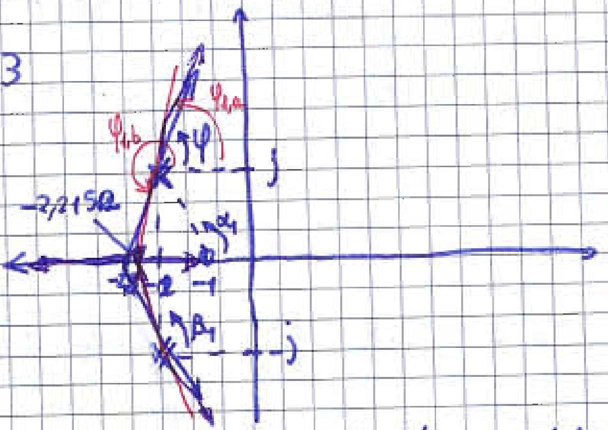
ESERCITAZIONE 8

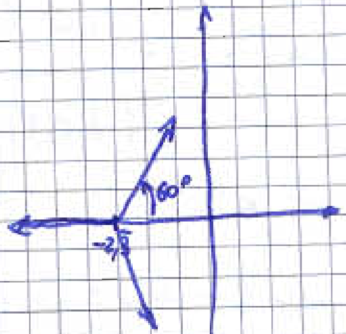
$$1 + K_1 \frac{s+1}{[(s+2)^2+1]^2}$$

trovo poli $(s+2)^2 + 1 = s^2 + 4s + 5 = 0$

$p_{1,2} = -2 \pm \sqrt{4-5} = -2 \pm j$ due poli di molt. 2.

$\rho = n - m = 4 - 1 = 3$





Trovo il punto d'intersezione degli asintoti

$$\sigma_a = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m} = \frac{-2+j-2+j-2-j-2-j+1}{3} = -\frac{7}{3} = -2,\bar{3}$$

Calcolo l'angolo d'inizio $(K_1 > 0)$

$$\underset{\substack{\uparrow \\ \text{molt. poli}}}{2}\varphi_1 = \pi + \underbrace{\sum_{j=1}^{m} \arg(p_i - z_i)}_{\alpha_1} - \underbrace{\sum_{i \neq j} \arg(p_i - p_j)}_{\beta_1} \mod 2\pi$$

$$2\varphi_1 = \pi + \left(\frac{\pi}{4} + \frac{\pi}{2}\right) - \underset{\substack{\uparrow \\ \text{molt. del} \\ \text{polo sotto}}}{2}\left(\frac{\pi}{2}\right) \mod 2\pi \qquad 2\varphi_1 = \frac{3}{4}\pi \mod 2\pi$$

$$2\varphi_{1,a} = \frac{3}{4}\pi \implies \varphi_{1,a} = \frac{3}{8}\pi = 67,5°$$

$$2\varphi_{1,b} = \frac{3}{4}\pi + 2\pi \implies \varphi_{1,b} = \frac{3}{8}\pi + \pi = \frac{11}{8}\pi = 67,5° + 180°$$

Trovo l'intersezione in cui ci sono le radici multiple.

$$\sum_{i=1}^{n} \frac{1}{s-p_i} - \sum_{i=1}^{m} \frac{1}{s-z_i} = 0 \qquad \frac{1}{s+2-j} + \frac{1}{s+2-j} + \frac{1}{s+2+j} + \frac{1}{s+2+j} - \frac{1}{s+1} = 0$$

$$2\,\frac{s+2\cancel{-j}+s+2\cancel{+j}}{(s+2)^2+1} - \frac{1}{s+1} = 0 \qquad 4\,\frac{s+2}{(s+2)^2+1} - \frac{1}{s+1} = 0$$

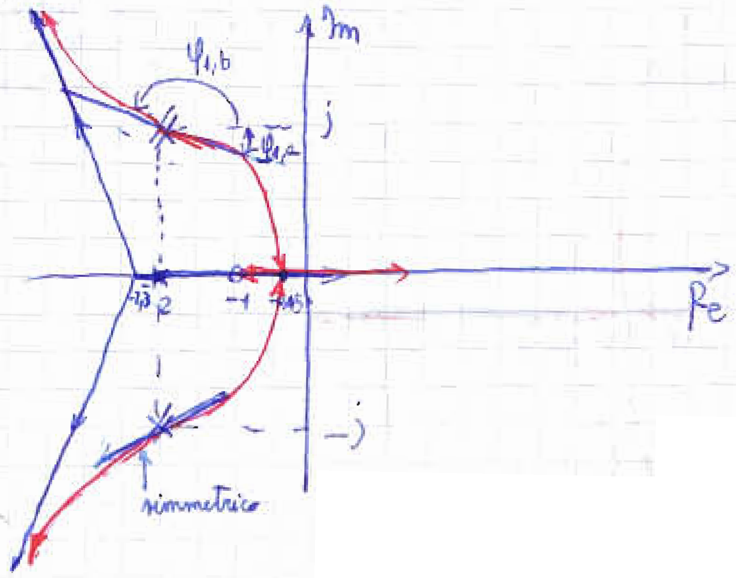
$$\frac{(4s+8)(s+1) - (s+2)^2 - 1}{\cancel{[(s+2)^2+1](s+1)}} = 0 \qquad 4s^2 + 12s + 8 - s^2 - 4s - 4 - 1 = 0$$

$$3s^2 + 8s + 3 = 0$$

$$s_{1,2} = \frac{-4 \pm \sqrt{16-9}}{3} = -\frac{4}{3} \pm \frac{\sqrt{7}}{3} = \begin{cases} -0,4514 \\ \underline{\underline{-2,2152}} \end{cases}$$

← questo per $K_1 < 0$ (luogo inverso)

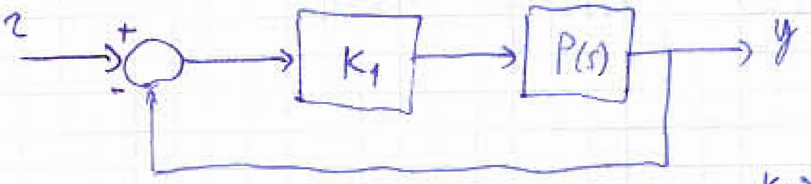
Luogo inverso ($K_1 < 0$)



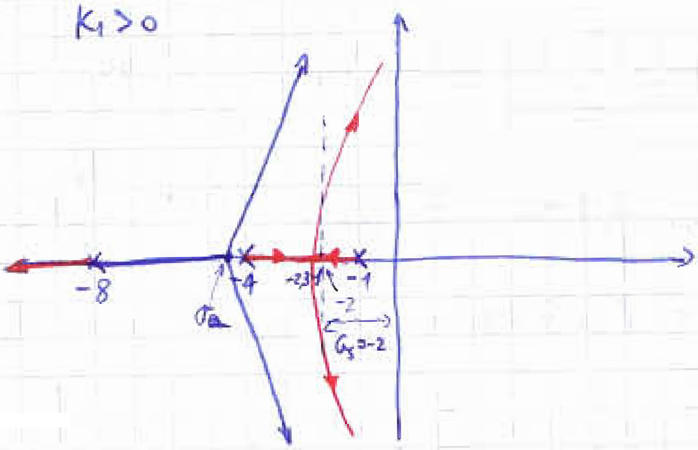
$$2\varphi_1 = \sum_{j=1}^{m} \arg(p_i - z_i) - \sum_{i \neq j} \arg(p_i - p_j)$$

$$2\varphi_1 = \frac{3}{4}\pi - 2\cdot\frac{\pi}{2} = -\frac{\pi}{4} \mod 2\pi$$

$$\varphi_{1,a} = -\frac{\pi}{8} \qquad \varphi_{1,b} = -\frac{\pi}{8} + \pi$$



$K_1 > 0$



$$P(s) = \frac{1}{(s+1)(s+4)(s+8)}$$

$p_1 = -1$

$p_2 = -4$

$p_3 = -8$

$\varrho = 3$ ↑ asintoti

$$\sigma_a = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{\varrho} = \frac{-1-4-8}{3} = -\frac{13}{3} = -4,\bar{3}$$

Se ci sono radici doppie, -4 e -1 prima si incontreranno poi andranno all'infinito.

$$\frac{1}{s+1} + \frac{1}{s+4} + \frac{1}{s+8} = 0 \qquad \frac{(s+4)(s+8)+(s+1)(s+8)+(s+1)(s+4)}{(s+1)(s+4)(s+8)} = 0$$

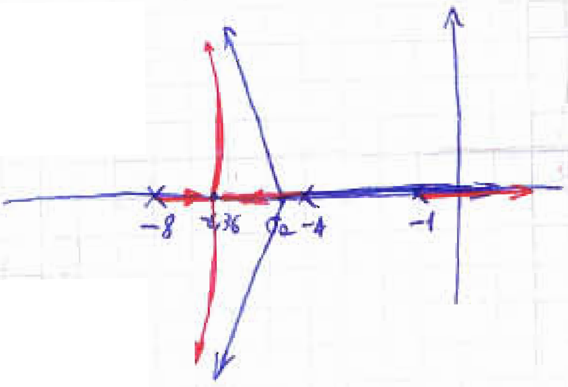
$$s^2 + 12s + 32 + s^2 + 9s + 8 + s^2 + 5s + 4 = 0$$

$$3s^2 + 26s + 44 = 0 \qquad s_{1,2} = \frac{-13 \pm \sqrt{169-132}}{3} = \frac{-13 \pm \sqrt{37}}{3} = \begin{cases} -6,36 \\ -2,31 \end{cases}$$

← perché deve essere tra -4 e -

Chi va su e chi va giù non importa.

$K_1 < 0$ LUOGO INVERSO



$$T_{ry} = \frac{K_1 P(s)}{1 + K_1 P(s)}$$

$$\frac{K_1 \cdot \frac{1}{\cancel{(s+1)(s+4)(s+8)}}}{\frac{(s+1)(s+4)(s+8)+K_1}{\cancel{(s+1)(s+4)(s+8)}}} = \frac{K_1}{(s+1)(s+4)(s+8)+K_1} \leftarrow \text{equazione caratteristica}$$

$$(s^2+5s+4)(s+8)+K_1 = s^3+8s^2+5s^2+40s+4s+32+K_1 = s^3+13s^2+44s+32+K_1 = 0$$

tabella di Routh

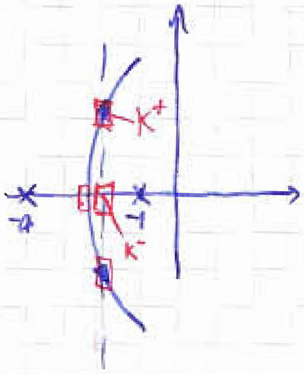
3	1	44	0	P	K_1: sistema asintoticamente stabile
2	13	$32+K_1$	0	P	$32+K_1>0 \Rightarrow K_1>-32$
1	$f(K_1)$	0	0	P	$f(K_1)>0 \quad \frac{-[(32+K_1)-44\cdot 13]}{13}>0$
0	$32+K_1$	0	0		

$540 - K_1 > 0 \qquad K_1 < 540$

Il sistema è asintoticamente stabile per $-32 < K_1 < 540$

$G_s \geq 2 \text{ sec}^{-1}$ grado di stabilità → tutti i poli hanno parte reale < -2. devono stare a sx di $Re = -2$

$K_1 \in [K^-, K^+]$



$s^3+13s^2+44s+32+K_1=0$

$z = s + G_s \Rightarrow z = s+2$

$Re(z)<0 \iff Re(s)+2<0 \Rightarrow Re(s)<-2 \qquad s = z-2$

$(z-2)^3+13(z-2)^2+44(z-2)+32+K_1 \Rightarrow z^3+7z^2+4z-12+K_1=0$

$$\begin{array}{llll} 3 & 1 & 4 & 0 \\ 2 & 7 & -12+K_1 & 0 \\ 1 & f(K_1) & 0 & \\ 0 & -12+K_1 & 0 & \end{array} \quad P \; P \; P$$

$-12+K_1>0 \qquad K_1>12$

$f(K_1)>0 \qquad -\left[\frac{-12+K_1-28}{7}\right]>0$

$40-K_1>0 \qquad K_1<40$

$K_1 \in \left]\underset{\underset{K^-}{\uparrow}}{12},\underset{\underset{K^+}{\uparrow}}{40}\right[$ sistema asintoticamente stabile

$G_s \geq 2 \iff K_1 \in [12,40]$ parentesi chiuse perchè $\geq$.

K_1 che massimizza G_s:

$K_1^* = \arg\max_{K \in \mathbb{R}} G_s(K_1)$ Il max ce l'ho quando -1 e -4 si incontrano

in $-2,31$: $\quad 1+K_1 G(s)=0 \qquad K_1 = -\frac{1}{G(s)} \Rightarrow K_1^* = -(s+1)(s+4)(s+8)\Big|_{s=-2,31} = 12,5972$

$1+3\cdot\frac{s+a}{(s+1)(s+2)}=0$ con $a>0$ luogo delle radici dipendente da a.

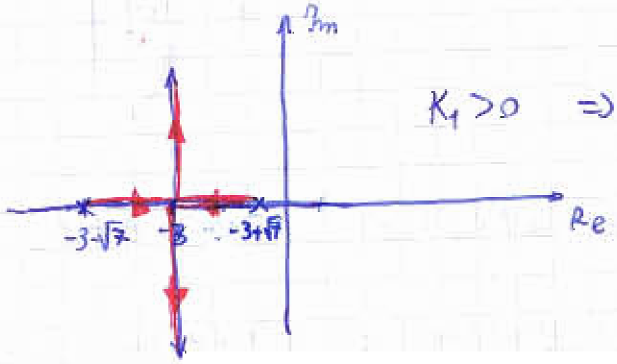
Devo togliere a da numeratore e usarlo come K.

$\frac{(s+1)(s+2)+3s+3a}{(s+1)(s+2)}=0 \qquad s^2+3s+2+3s+3a=0 \qquad \underbrace{s^2+6s+2}_{\text{divido per}}+3a=0$

$1+\overset{K_1}{3a}\cdot\frac{1}{s^2+6s+2}=0$ poli: $s^2+6s+2=0 \qquad p_{1,2}=-3\pm\sqrt{9-2}=-3\pm\sqrt{7}$

$\rho = n-m = 2$

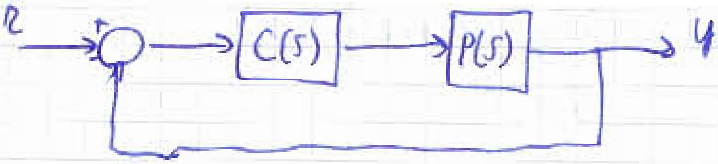
$K_1>0 \Rightarrow a>0$



$\sigma_a = \frac{-3+\sqrt{7}-3-\sqrt{7}}{2} = -3$

ESERCITAZIONE 9

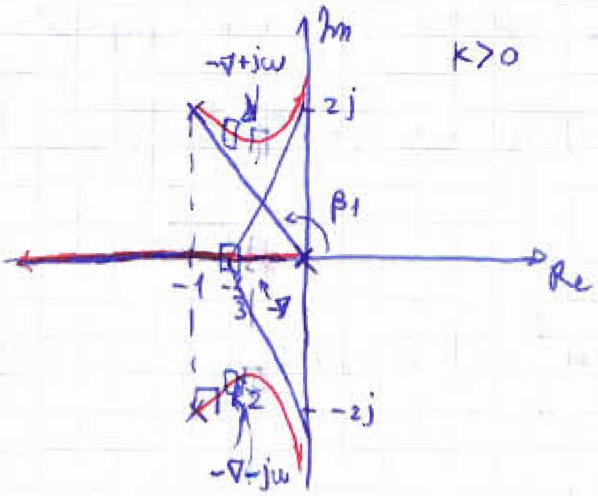
21/05/09



$P(s) = \dfrac{1}{s(s^2+2s+5)}$ $\quad C(s) = K \in \mathbb{R}$ affinchè K max

$p_{1,2} = \dfrac{-1 \pm \sqrt{1-5}}{1} = -1 \pm j2$

$p_3 = 0$ $\quad \rho = 3$ n° asintoti



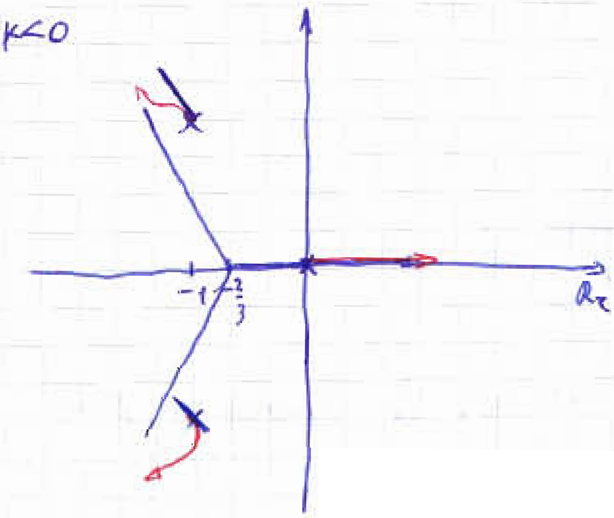
$$\sigma_a = \frac{\sum p_i - \sum z_i}{\rho} = \frac{-1+j2-1-j2+0}{3} = -\frac{2}{3}$$

$$\varphi_1 = \pi + \sum_{j=1}^{m} \arg(p_i - z_j) - \sum_{i \neq j} \arg(p_i - p_j)$$

base = 1
altezza = 2

$$\beta_1 = \frac{\pi}{2} + \frac{\pi}{6} \qquad \beta_2 = \frac{\pi}{2}$$

$$\varphi_1 = \pi + \left(\frac{\pi}{2} + \frac{\pi}{6}\right) - \frac{\pi}{2} = -\frac{\pi}{6}$$



$$\varphi_1 = -\frac{\pi}{2} - \frac{\pi}{6} - \frac{\pi}{2} = -\frac{7}{6}\pi$$

K<0 SISTEMA INSTABILE perchè il polo in 0 va subito nella parte di piano Re>0

Per K>0 il massimo di G_s a l'ho quando tutti e tre i poli sono allineati.

Uso il teorema del baricentro.

$\sum p_i = \text{cost.}$ $\qquad -\nabla-\nabla+\cancel{j\omega}-\nabla-\cancel{j\omega}=0-1+\cancel{2j}-1-\cancel{2j}$

$-3\nabla=-2 \qquad \nabla=\frac{2}{3}$

I tre poli valgono

$\boxed{-\frac{2}{3}}\ ,\ -\frac{2}{3}+j\omega,\ -\frac{2}{3}-j\omega$

l'unico che conosco in modo completo.

$1+KP(s)=0 \implies 1+K^*\cdot \underset{\underset{\text{polo}}{\uparrow}}{P\left(-\frac{2}{3}\right)}=0 \qquad K^*=-\frac{1}{P\left(-\frac{2}{3}\right)}=-\dot{S}\left(S^2+2S+5\right)\Big|_{S=-\frac{2}{3}}=$

$K=-\frac{1}{P(s)}$

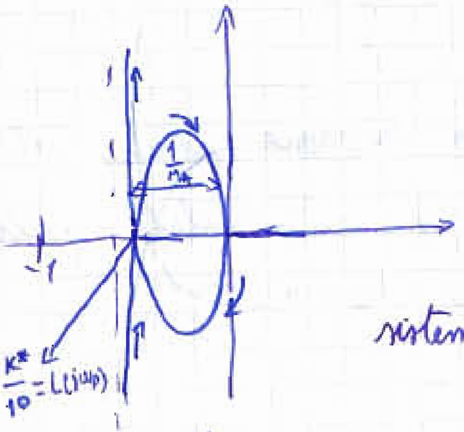
$=\frac{2}{3}\left(\frac{4}{9}-\frac{4}{3}+5\right)\simeq 2{,}74 \qquad C(s)=2{,}74$

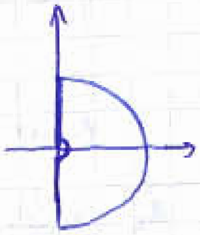
$P(s)=\frac{1}{S(S+1-2j)(S+1+2j)} \qquad L(j\omega)=\frac{1}{j\omega(j\omega+1-2j)(j\omega+1+2j)}$

$|L(j\omega)|=\frac{1}{|\omega|\sqrt{1+(\omega-2)^2}\sqrt{1+(\omega+2)^2}} \qquad \arg L(j\omega)=-\frac{\pi}{2}-\operatorname{arctg}(\omega-2)-\operatorname{arctg}(\omega+2)$

$\omega\to 0^+ \quad |L(j\omega)|=+\infty \qquad \omega\to+\infty \quad |L(j\omega)|=0$

$\arg L(j\omega)=-\frac{\pi}{2} \qquad \arg L(j\omega)=-\frac{3}{2}\pi$





sistema stabile ⇒ -1 non lo circondo o tocco

$M_A=\frac{1}{|L(j\omega_p)|} \qquad \omega_p$ t.c. $\arg L(j\omega_p)=-\pi$

Calcolo ω_p con Routh

$1+\alpha L(s)=0$ devo trovare α tale che le radici siano puramente immaginarie

$$L(s)=-\frac{1}{\alpha} \Rightarrow L(\pm j\omega)=-\frac{1}{\alpha}=L(j\omega_p) \qquad 1+\overset{\beta}{\boxed{\alpha\cdot K^*}}\cdot\frac{1}{s(s^2+2s+5)}=0$$

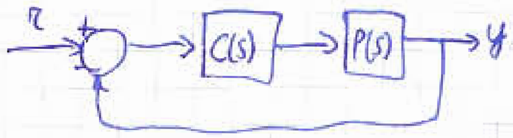
$$s(s^2+2s+5)+\beta=0 \qquad s^3+2s^2+5s+\beta=0$$

$$\begin{array}{c|ccc} 3 & 1 & 5 & 0 \\ 2 & 2 & \beta & 0 \\ 1 & f(\beta) & 0 & \\ 0 & \beta & 0 & \end{array} \qquad f(\beta)=-\frac{1}{2}\left(\beta-10\right)=0 \qquad \beta=10$$

la voglio composta da tutti zeri → singolarità completa → radici puramente immaginarie perchè pol. ausiliario di grado 2 → 2 radici simmetriche rispetto all'origine.

$$\alpha K^*=10 \qquad -\frac{1}{\alpha}=-\frac{K^*}{10} \qquad M_A=\frac{1}{|L(j\omega_p)|}=\frac{10}{K^*}=\frac{10}{2{,}74}=3{,}65$$

②



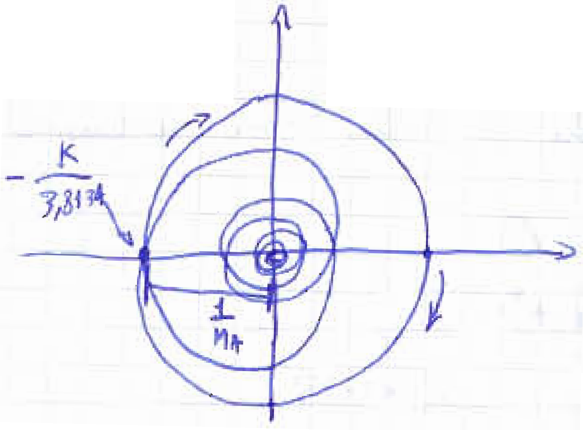
$$C(s)=K>0 \in \mathbb{R} \qquad P(s)=\frac{e^{-2s}}{1+4s}$$

$$1+KP(s)=0 \qquad L(s)=Ke^{-2s}\cdot\frac{1}{1+4s} \qquad L(j\omega)=Ke^{-2j\omega}\cdot\frac{1}{1+4j\omega}$$

$$|L(j\omega)|=K\cdot 1\cdot\frac{1}{\sqrt{1+16\omega^2}} \qquad \arg L(j\omega)=-2\omega-\operatorname{arctg}4\omega$$

$$\omega\to 0^+ \quad |L(j\omega)|=K \qquad \omega\to+\infty \quad |L(j\omega)|=0$$

$$\arg L(j\omega)=0 \qquad \arg L(j\omega)=\infty$$



$$\arg L(j\omega_p) = -\pi \Rightarrow \underbrace{+2\omega_p + \text{arctg}\,4\omega_p}= +\pi$$

$\omega_p = 1 \quad 3{,}3258 > \pi$

$\omega_p = 0{,}9 \quad 3{,}0998 < \pi$

$\omega_p = 0{,}92 \quad 3{,}1455 \simeq \pi$

$$|L(j\omega_p)| = \frac{K}{\sqrt{1+16\omega_p^2}} = \frac{K}{3{,}8134}$$

Il sistema è stabile quando $-\frac{K}{3{,}8134} > -1 \quad 0<K<3{,}8134$

Il sistema è asintoticamente stabile per $K \in \left]0; 3{,}8134\right[$

Trovare K tale che $M_A = 2$

$$M_A = \frac{1}{\frac{K}{3{,}8134}} = 2 \qquad 2K = 3{,}8134 \qquad K = 1{,}9067$$

$T_a \approx \frac{3}{G_s}$ Calcolo G_s per $K = 1{,}9067$

approssimanti di Padé: $e^{-t_0 s} = \frac{1-\frac{t_0}{2}s}{1+\frac{t_0}{2}s} \qquad e^{-2s} = \frac{1-s}{1+s}$

$$T_{ry}(s) = \frac{K\frac{e^{-2s}}{1+4s}}{1+\frac{Ke^{-2s}}{1+4s}} \simeq \frac{K\cdot\frac{1-s}{1+s}\cdot\frac{1}{1+4s}}{1+\underset{1{,}9067}{K}\cdot\frac{1-s}{1+s}\cdot\frac{1}{1+4s}}$$

denominatore $\rightarrow 4s^2+3{,}0933s+2{,}9067=0$

$$s_{1,2} = \underbrace{-0{,}3867}_{G_s} \pm j0{,}7597$$

$$T_a = \frac{3}{0{,}3867} = 7{,}75 \text{ sec}$$

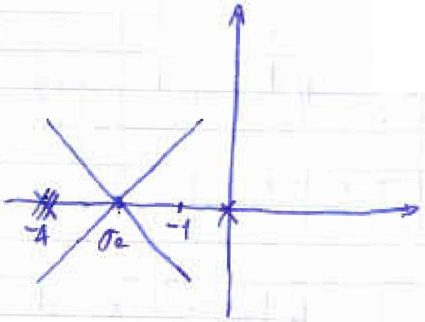
④ $C(s) = K > 0$

$$P(s) = \frac{1}{s(s+4)^3}$$

$p_1 = 0$

$p_{2,3,4} = -4$

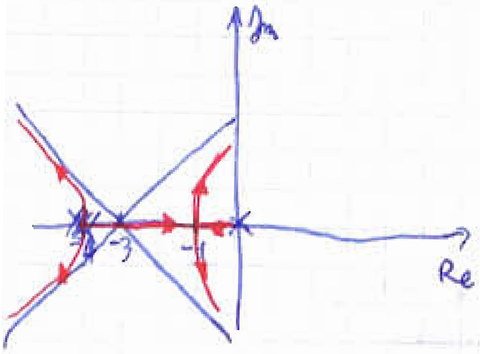
$\rho = n - m = 4$



$$\sigma_a = \frac{0-4-4-4}{4} = -3$$

radici doppie $\sum_i \frac{1}{s-p_i} - \sum_i \frac{1}{s-z_i} = 0$ $\quad \frac{1}{s} + \frac{3}{s+4} = 0 \quad s+4+3s=0 \quad s=-1$

Angolo di partenza 0 perché tutti allineati



$$T_{ry}(s) = \frac{L(s)}{1+L(s)} = \frac{\frac{K}{s(s+4)^3}}{1+\frac{K}{s(s+4)^3}} = \frac{K}{s(s+4)^3+K}$$

$$s^4+12s^3+48s^2+64s+K=0$$

4	1	48	K	0
3	12	64	0	0
2	$\frac{512\ 128}{12\ 3} \to 3K$		0	0
1	$2048-9K$	0	0	
0	$3K$	0		

stabilità asintotica

$3K > 0$ verificata

$2048 - 9K > 0 \qquad K < \frac{2048}{9} \simeq 227,5$

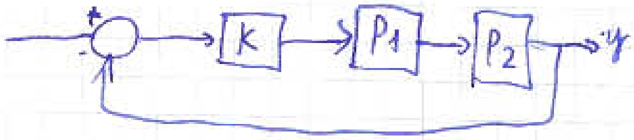
Il sistema è as. stabile per $K \in]0;227,5[$

PUNTO 3) Vuol dire che le radici del polinomio caratteristico devono essere puramente immaginarie

$2048 - 9K = 0 \qquad K = \frac{2048}{9} \qquad a_2(s) = 128 s^2 + 3K = 0$

$s_{1,2} = \pm j\,2{,}309$

COMPITINO 2005 25/02/09



$K \in \mathbb{R}$

$P_1(s) = \dfrac{1}{s+6}$ $\qquad L(s) = K P_1 P_2 = K \cdot \dfrac{s+10}{s(s+3)(s+6)}$

$P_2(s) = \dfrac{s+10}{s(s+3)}$ $\qquad 1+L(s)=0 \qquad s^3+9s^2+(18+K)s+10K=0$

Per quali K è

- as. stabile
- instabile

$$\begin{array}{c|ccc} 3 & 1 & 18+K & 0 \\ 2 & 9 & 10K & 0 \\ 1 & 162-K & 0 & \\ 0 & 10K & & \end{array}$$

as. stabile se $\begin{cases} 162-K>0 \\ 10K>0 \end{cases}$ $\quad K \in (0,162) \quad \Rightarrow \quad K<0$ o $K>162$ INSTABILE

Analizzo i punti:

$K=0$
$$\begin{array}{c|cc} 3 & 1 & 18 \\ 2 & 9 & 0 \\ 1 & 162 & \\ 0 & 0 & \end{array}$$

← eq. ausiliarie (riga 1); riga di 0 (riga 0)

$a_2(s) = 162s = 0 \Rightarrow \overset{\text{molt. } 1}{s=0} \in \mathrm{Im}.$

$\Rightarrow$ permanenza

$K=162$
$$\begin{array}{c|cc} 3 & 1 & 180 \\ 2 & 9 & 1620 \\ 1 & 0 & 0 \\ 0 & & \end{array}$$

$\leftarrow a_2(s) = 9s^2 + 1620. = 0 \qquad s = \pm j\sqrt{13} \in \mathrm{Im}$

$\Rightarrow$ stabile $\quad K \in [0,162]$

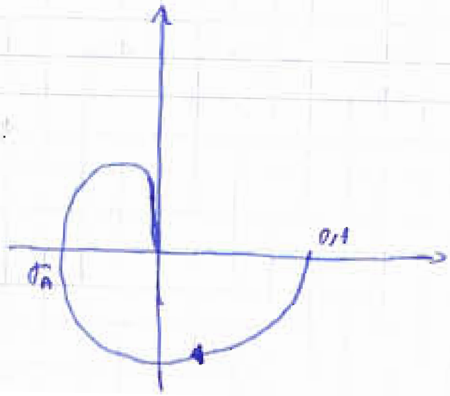
$$G(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)(j\omega+5)}$$ diagramma di Nyquist semplice

$$\arg G(j\omega) = -\text{arctg}\, \omega - \text{arctg}\, \frac{\omega}{2} - \text{arctg}\, \frac{\omega}{5}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\omega^2+1}\ \sqrt{\omega^2+4}\ \sqrt{\omega^2+25}}$$

$\omega \to 0 \quad |G(j\omega)| \to \frac{1}{10} \quad \arg G(j\omega) \to 0$

$\omega \to +\infty \quad |G(j\omega)| \to 0 \quad \arg G(j\omega) \to -\frac{3}{2}\pi$



$$\sigma_A : \quad -\text{arctg}\, \omega - \text{arctg}\, \frac{\omega}{2} - \text{arctg}\, \frac{\omega}{5} = -\pi$$

$$+\text{arctg}\, \frac{\omega}{5} + \text{arctg}\, \frac{\omega}{2} = +\pi - \text{arctg}\, \omega$$ applico $tg(\alpha \pm \beta)$

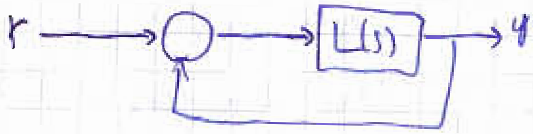
$$\frac{\frac{\omega}{2} + \frac{\omega}{5}}{1 - \frac{\omega^2}{10}} = \frac{0-\omega}{1+0} \qquad -\omega = \frac{\frac{7}{\cancel{10}}\omega}{\frac{10-\omega^2}{\cancel{10}}}$$

$$\omega^3 - 10\omega = 7\omega \qquad \omega^3 - 17\omega = 0 \qquad \omega(\omega^2 - 17) = 0 \qquad \omega = 0 \text{ non acc.}$$

$$\omega = \sqrt{17}$$

$$|G(j\omega)|_{\omega=\sqrt{17}} = 7{,}9 \cdot 10^{-3} = \sigma_A$$

stabile



$$L(s) = \frac{7}{(s+1)(s+9)}$$

$$T_{ry}(j\omega) = \frac{7}{(j\omega+2)(j\omega+8)} = \frac{L(s)}{1+L(s)} = \frac{7}{2}\ \frac{1}{(1+\frac{j\omega}{2})\,8\,(1+\frac{j\omega}{8})}$$

$\omega = 2$

$$\begin{bmatrix} y(t) = ? \\ r(t) = 5 \cdot 1(t) \end{bmatrix}$$

$$|T_{ry}(j\omega)| = \frac{7}{\sqrt{\omega^2+4}\ \sqrt{\omega^2+64}} \simeq 0{,}3$$

$y(t) = ?$ per $t \to +\infty$ se $r(t) = 3 \,\text{sen}(\underset{\uparrow \atop \omega}{2}t) \cdot 1(t)$

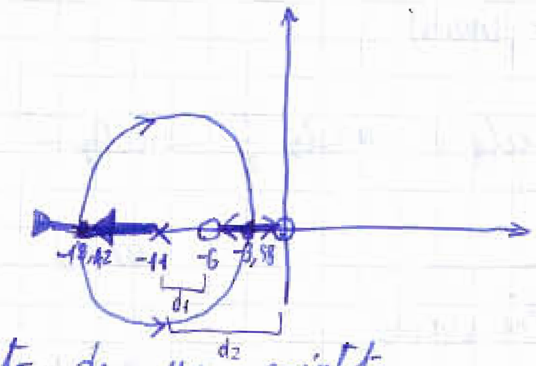
teorema analisi armonica

$$\arg T_{ry}(j\omega) = -\text{arctg}\, \frac{\omega}{2} - \text{arctg}\, \frac{\omega}{8} = -\text{arctg}\, 1 - \text{arctg}\, \frac{1}{4} \simeq -1{,}03$$

$$y(t)_\infty = 3 \cdot |T_{ry}(j2)|\ \text{sen}\,(2t + \arg T_{ry}(j2)) = 0{,}9\ \text{sen}\,(2t - 1{,}03)$$

$$1 + 10 \frac{1+\tau s}{(1+2\tau s)(1+s)} = 0 \quad \tau > 0$$ trovare luogo radici

$$1+\underbrace{2\tau}_{k>0} \frac{s(s+6)}{s+11} = 0 \quad \underline{\underline{m>n}}$$



Il polo andrà in uno 0

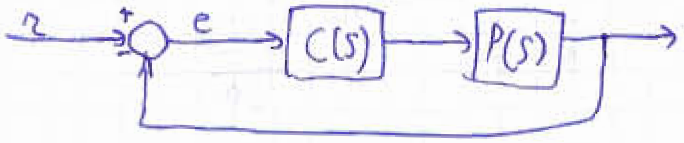
Il restante zero sarà raggiunto da un asintoto

$$\frac{1}{s+11} - \frac{1}{s} - \frac{1}{s+6} = 0$$ $s_1 = -3,58$; $s_2 = -18,42$

radici doppie → tante frecce entrano, tante escono

$$R = \sqrt{d_1 d_2} = \sqrt{5,11}$$

28/05/09



$$P(s) = \frac{10}{(s+1)(s+5)}$$ progettare $C(s)$ in modo che $\underset{\substack{\uparrow \\ \text{errore a regime}}}{e_r} = 0$

$$e_r = \frac{1}{1+\underset{\substack{\uparrow \\ \text{costante di posizione}}}{k_p}}$$ $k_p = L(0)$ ⇒ $L(0) \to +\infty$ ⇒ polo nell'origine nel controllore

⇓

$C(s)$ almeno di ordine 1.

$$C(s) = K \cdot \frac{s+b}{s}$$ fare $\underline{\text{sempre}}$ controllori con $n = m$ perchè lo 0 fa comodo

$$L(s) = k \cdot \frac{s+b}{s} \cdot \frac{10}{(s+1)(s+5)}$$ $L(0) = +\infty$ $e_r = 0$ $\underline{\text{OK}}$

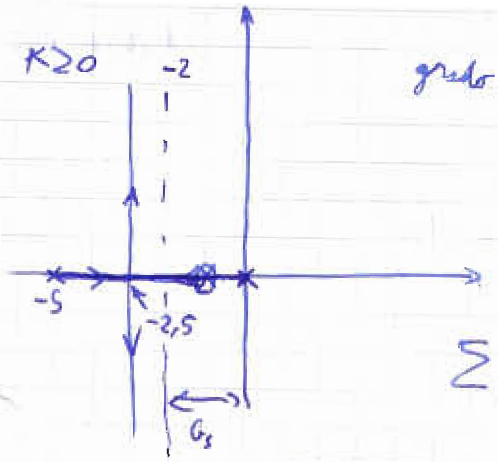
$T_a \leq 1,5\ s$ $S = 0\%$ ⇒ faccio il luogo delle radici

$\uparrow$ $\frac{3}{G} \Rightarrow G_s \geq 2$

⇓ tutti i poli reali

Determino b subito per eliminare un polo: quello più vicino all'asse immaginario → più instabile.

$b = +1$ $\quad L(s) = 10K \dfrac{1}{s(s+5)}$ luogo delle radici:



grado uguale a 2
$\uparrow$
asintoti

Cerco le radici doppie

$$\sum \frac{1}{s-p_i} - \sum \frac{1}{s-z_i} = \frac{1}{s} + \frac{1}{s+5} = 0 \Rightarrow s+5+s=0 \Rightarrow s = -\frac{5}{2} = -2{,}5$$

Devo stare a sinistra di -2 (per $\sigma_s \geq 2$) e prima di -2.5 (per $S=0$), altrimenti i poli si allontanano e non ho $S=0$. $K \in [K^-, K^+]$

$$T_{ry}(s) = \frac{L(s)}{1+L(s)} = \frac{10K \cdot \frac{1}{s(s+5)}}{1+10K\frac{1}{s(s+5)}} \qquad \underset{\substack{\uparrow \\ \text{polinomio} \\ \text{caratteristico}}}{p_c(s)} = s(s+5)+10K = s^2+5s+10K = 0$$

$$p_c(-2) = (-2)^2 + 5(-2) + 10K = 0 \qquad K = 0{,}6 \qquad C(s) = 0{,}6 \cdot \frac{s+1}{s}$$

Margine di fase:

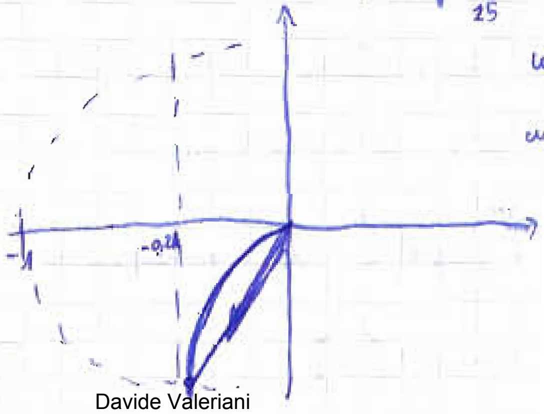
$$L(s) = C(s)P(s) = 0{,}6 \cdot \frac{\cancel{s+1}}{s} \cdot 10 \cdot \frac{1}{(\cancel{s+1})(s+5)} = 6 \cdot \frac{1}{s(s+5)}$$

$$L(j\omega) = \frac{6}{5} \cdot \frac{1}{j\omega\left(1+\frac{j\omega}{5}\right)} \qquad |L(j\omega)| = \frac{6}{5} \cdot \frac{1}{|\omega| \cdot \sqrt{1+\frac{\omega^2}{25}}} \qquad \arg L(j\omega) = -\frac{\pi}{2} - \operatorname{arctg}\frac{\omega}{5}$$

$$\sigma_a = \frac{6}{5} \cdot \left(-\frac{1}{5}\right) = -0{,}24$$

$\omega \to 0$ $\quad |L(j\omega)| = +\infty$ $\quad \arg L(j\omega) = -$

$\omega \to \infty$ $\quad |L(j\omega)| = 0$ $\quad \arg L(j\omega) = -$



$M_F = \pi - \arg(L(j\omega_c)) \qquad \omega_c \Rightarrow |L(j\omega_c)| = 1$

$$\frac{6}{5} \cdot \frac{1}{|\omega| \sqrt{1+\frac{\omega^2}{25}}} = 1 \Rightarrow \frac{6}{5} = |\omega| \sqrt{1+\frac{\omega^2}{25}} \Rightarrow 6 = |\omega| \sqrt{25+\omega^2}$$

$$36 = \omega^2(\omega^2+25) \qquad x = \omega^2 \qquad x^2+25x-36=0 \qquad x = \frac{-25 \pm \overbrace{\sqrt{25^2+4 \cdot 36}}^{\sqrt{769}}}{2} = \begin{cases} \frac{-25-\sqrt{769}}{2} < 0 \ \text{N.A.} \\ 1{,}3654 \ \text{ACC.} \end{cases}$$

$$\omega_c = \sqrt{1{,}3654} = 1{,}1685 \ \frac{rad}{s}$$

$$\arg L(j\omega_c) = -\frac{\pi}{2} - \operatorname{arctg} \frac{1{,}1685}{5} = \dots \qquad M_F = \pi - \dots = 76{,}8°$$

ESERCITAZIONE 10

④ $P(s) = \dfrac{10}{(s+2)(s+5)(s+10)} \qquad C(s) = K$ con $K \in \mathbb{R} \qquad e_r = 0{,}05$

$$e_r = \frac{1}{1+K_p} \qquad K_p = L(0) = K \left. \frac{10}{(s+2)(s+5)(s+10)} \right|_{s=0} = K \cdot \frac{1\cancel{0}}{10\cancel{0}} = \frac{K}{10}$$

$$e_r = \frac{1}{1+\frac{K}{10}} = 0{,}05 \qquad \frac{10}{10+K} = 0{,}05 \qquad 10 = 0{,}5 + 0{,}05K \qquad K = \frac{9{,}5}{0{,}05} = 190$$

Verifico che il sistema sia stabile

$$\underset{\text{eq. car.}}{1+L(s)} = 1 + \frac{10K}{(s+2)(s+5)(s+10)} = 0 \qquad (s^2+7s+10)(s+10) + 10\overset{190}{K} = 0$$

$$s^3 + 17s^2 + 80s + 2000 = 0$$

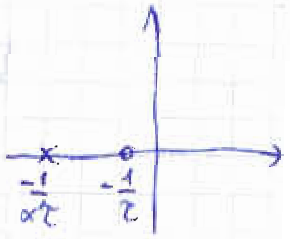
$$\begin{array}{c|ccc} 3 & 1 & 80 & 0 \\ 2 & 17 & 2000 & 0 \\ 1 & f(s) & 0 & \\ 0 & 2000 & & \end{array} \qquad f(s) = (17 \cdot 80 - 2000) \cdot \frac{1}{17} = -640$$

$P \quad V \quad V$

$\Rightarrow$ sistema non stabile $\Rightarrow$ non è possibile progettare un controllore ...

- $e_r = 0{,}05 \qquad M_a = 2$. progettare rete anticipatrice

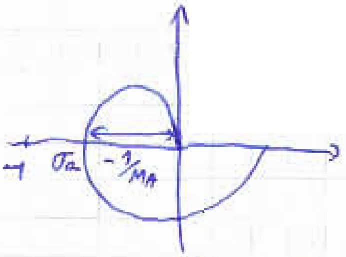
$$C(s) = K \cdot \frac{1+\tau s}{1+\alpha\tau s} \quad \text{con } \alpha \in [0,1]$$



Pongo $-\frac{1}{\tau} = 2$ per eliminare il polo che dà più fastidio.

$$C(s) = \underset{\overset{''}{190}}{K} \cdot \frac{1+\frac{1}{2}s}{1+\alpha\cdot\frac{s}{2}}$$

Per trovare α uso $M_A = 2$



σ_a deve essere $-\frac{1}{2}$ uso $1+\alpha_c L(s)=0$ imponendo le radici immaginarie

$$L(\pm j\omega) = -\frac{1}{\alpha_c} = -\frac{1}{2} \Rightarrow \alpha_c = 2 \qquad 1+2L(s)=0 \qquad \frac{1}{2}+L(s)=0$$

$$L(s) = 190 \cdot \frac{1+\frac{1}{2}s}{1+\alpha\frac{1}{2}s} \cdot \frac{10}{(s+2)(s+5)(s+10)} = 1900 \cdot \frac{\cancel{s+2}}{\alpha s+2} \cdot \frac{1}{\cancel{(s+2)}(s+5)(s+10)} = \frac{1900}{(\alpha s+2)(s+5)(s+10)}$$

$$\frac{1}{2} + \frac{1900}{(\alpha s+2)(s+5)(s+10)} = 0 \qquad \frac{1900}{(\alpha s+2)(s+5)(s+10)} = -\frac{1}{2}$$

$$3800 = -(\alpha s+2)(s+5)(s+10) \qquad \alpha s^3 + (15\alpha+2)s^2 + (50\alpha+30)s + 3900 = 0$$

Cerco radici puramente immaginarie $\Rightarrow$ Routh

$$\begin{array}{llll} 3 & \alpha & 50\alpha+30 & 0 \\ 2 & 15\alpha+2 & 3900 & 0 \\ 1 & f(\alpha) & 0 & 0 \\ 0 & 50\alpha+30 & 0 & \end{array}$$

$$f(\alpha) = \frac{(15\alpha+2)(50\alpha+30) - 3900\alpha}{15\alpha+2}$$

voglio che $f(\alpha)=0$

$$750\alpha^2 + 550\alpha + 60 - 3900\alpha = 0 \qquad \alpha_{1,2} = \begin{cases} 4{,}4487 \\ 0{,}0180 \leftarrow \end{cases}$$

perchè $\alpha \in (0,1)$

perchè sto cercando un valore di ω tale che $\sigma_a = -\frac{1}{2}$

$$C(s) = 190 \cdot \frac{1+\frac{1}{2}s}{1+0{,}0180\cdot\frac{1}{2}s}$$

② $P(s) = \frac{1}{s^2}$ $\qquad \underset{\text{ordine 1}}{C(s)} = \frac{bs+c}{s+a} = K \cdot \frac{s+\beta}{s+\alpha}$ $\qquad 1+L(s)=0$

$$1 + \frac{bs+c}{s^2(s+a)} = 0$$

$s^2(s+a)+bs+c=0$ $\quad s^3+as^2+bs+c=0 \Leftarrow P_c(s)$ voglio poli in $-1,-2,-4$

$$\parallel$$

$P_d(s)=(s+1)(s+2)(s+4)=(s^2+3s+2)(s+4)=s^3+7s^2+14s+8$

$\Rightarrow a=7$

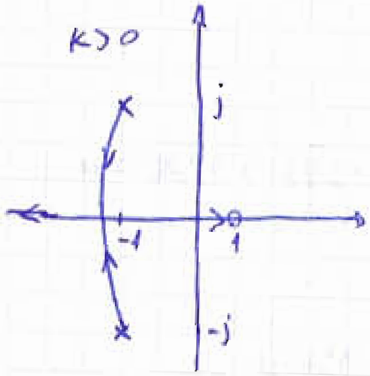
$b=14$ $\quad C(s)=\frac{14s+8}{s+7}=14\cdot\frac{s+0,57}{s+7}$

$c=8$

① $P(s)=\frac{s-1}{s^2+2s+2}$ $\quad e_r=0$ $\quad T_a=3s$ $\quad S=0\%$ $\quad p_1=-1+j$ $\quad z_1=1$

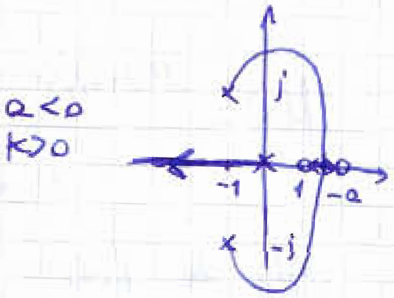
$\searrow G_s=1$ $\quad p_2=-1-j$

1) $C(s)=k \in \mathbb{R}$ $\quad 1+k\frac{s-1}{s^2+2s+2}=0$

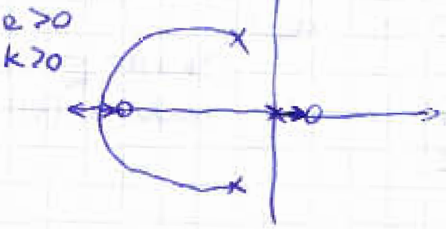


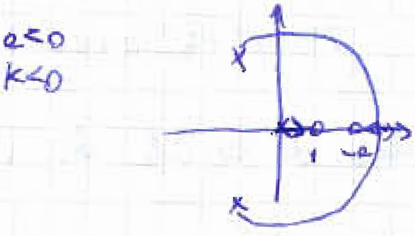
$e_r=0 \Rightarrow$ polo nell'origine $\Rightarrow$ regolatore ordine 0 inutile.

2) $C(s)=k\frac{s+a}{s}$

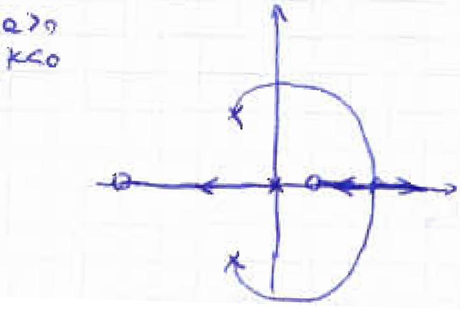


G_s mai 1 perchè impossibile avere tutti e tre i poli allineati in -1.





instabile



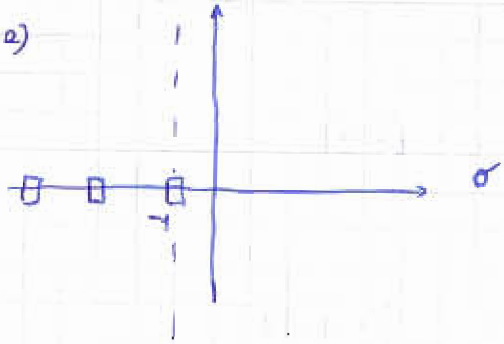
altro modo: algebrico

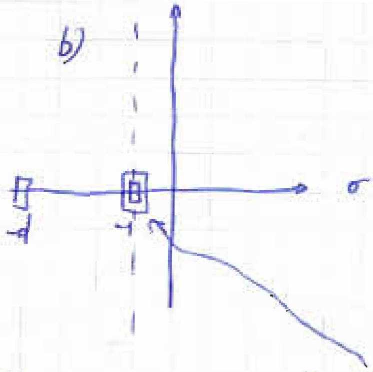
$C(s)=\frac{b_1s+b_0}{s}$ $\quad 1+\frac{b_1s+b_0}{s}\cdot\frac{s-1}{s^2+2s+2}=0$

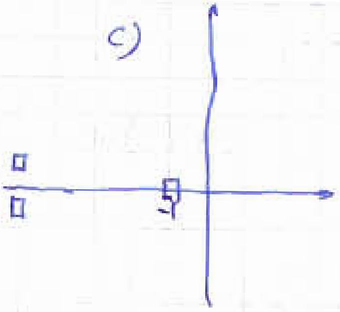
pol. car.: $P_c(s) = s(s^2+2s+2) + (b_1 s + b_0)(s-1) = s^3+2s^2+2s+b_1s^2+(b_0-b_1)s - b_0 =$

$= s^3 + (2+b_1)s^2 + (2-b_1+b_0)s - b_0 = 0$

Imposto il polinomio caratteristico







b) $P_c(s) = s^3 + (2+b_1)s^2 + (2-b_1+b_0)s - b_0$ $\quad$ $Pd(s) = (s+1)^2(s-d)$ $\quad$ $d \leq -1$

$Pd(s) = s^3 + (2+d)s^2 + (2d+1)s + d$

$$\begin{cases} 2+b_1 = 2+d \Rightarrow b_1 = d \\ 2-b_1+b_0 = 2d+1 \Rightarrow -d-d+2 = 1+2d \Rightarrow d = \frac{1}{4} \\ -b_0 = d \Rightarrow b_0 = -d \\ d \leq -1 \end{cases} \quad \emptyset$$

a) $pd(s) = (s+1)(s^2+a_1 s + a_0) = s^3 + (a_1+1)s^2 + (a_1+a_0)s + a_0$ $\quad$ $Re\{s_{1,2}\} \leq -1$

$s^2 + a_1 s + a_0 = 0$ $\quad$ cambio di variabile $\quad$ $z = s+1 \rightarrow Re\{z\} \leq 0$ $\quad$ $(z-1)^2 + a_1(z-1) + a_0 = 0$

$s = z-1$

$\Rightarrow z^2 + (a_1-2)z + a_0 - a_1 + 1 = 0$ $\quad$ $\begin{cases} a_1 - 2 > 0 \\ a_0 - a_1 + 1 > 0 \end{cases}$ per essere radici a parte reale positiva

$$\begin{cases} 2+b_1 = a_1+1 \\ 2-b_1+b_0 = a_1+a_0 \\ -b_0 = a_0 \\ a_1 > 2 \\ a_0 > a_1 - 1 \end{cases} \qquad \begin{cases} b_1 = a_1 - 1 \\ b_0 = -a_0 \\ 2 - a_1 + 1 - a_0 = a_1 + a_0 \longrightarrow a_1 = \frac{3}{2} - a_0 \rightarrow a_0 = \frac{3}{2} - a_1 \\ a_1 > 2 \\ a_0 > a_1 - 1 \longrightarrow a_1 < a_0 + 1 \end{cases} \Rightarrow \begin{cases} a_1 > 2 \\ a_1 < \frac{3}{4} \end{cases} \quad \emptyset$$

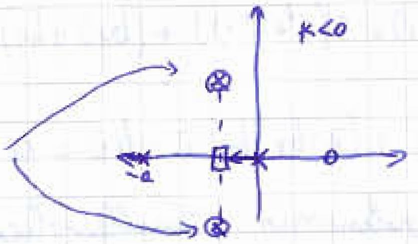
$\Rightarrow$ Controllore di ordine 2.

04/06/09

$T_a \simeq 3\,s \qquad e_r = 0 \qquad S = 0\,\%$

$$C(s) = K \cdot \frac{(s+b_1)(s+b_2)}{\underset{\nearrow}{s}(s+a)} = K \cdot \frac{s^2+\beta_1 s+\beta_2}{s(s+a)}$$

errore a regime 0



Metto gli o sopra ai poli

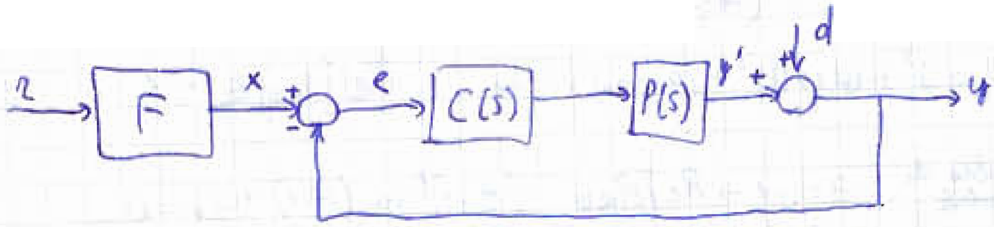
$\beta_1 = \beta_2 = 2$

$$G_s = 1 \qquad \underset{\substack{\downarrow \\ C(s)P(s)}}{1 + L(s) = 0} \qquad 1 + K \cdot \frac{s-1}{s(s+a)} = 0 \qquad s^2 + (a+K)s - K = 0$$

$$P_d(s) = (s+1)\underset{\uparrow}{(s+10)} = s^2 + 11s + 10 \qquad \begin{cases} a + K = 11 \\ -K = 10 \end{cases} \qquad \begin{cases} a = 21 \\ K = -10 \end{cases}$$

a caso basta che sia dominante

$$C(s) = -10 \cdot \frac{s^2+2s+2}{s(s+21)}$$



$F \in \mathbb{R}$

$d(t) = 4\,\mathrm{sen}(\underset{\substack{\uparrow \\ \omega_d}}{3}t)$ Determinare C(s) di ordine minimo.

$$y' = CPe \qquad y = y' + d = CPe + d \qquad e = x - y$$

$$T_{dy}(s) = \frac{1}{1 + C(s)P(s)} \qquad y = CP(x-y) + d \;\Rightarrow\; Y(s) = \underbrace{\frac{C(s)\cdot P(s)}{1 + C(s)P(s)}}_{T_{xy}(s)} X + \underbrace{\frac{1}{1 + C(s)P(s)}}_{T_{dy}(s)} D$$

Voglio che $T_{dy}(s)$ si annulli per $\omega_d = 3$

$$C(s) = \frac{n_c(s)}{d_c(s)} \qquad P(s) = \frac{n_p(s)}{d_p(s)} \qquad T_{dy} = \frac{1}{1 + \frac{n_c(s)\,n_p(s)}{d_c(s)\,d_p(s)}} = \frac{d_c\,d_p}{d_c\,d_p + n_c\,n_p}$$

Voglio dei poli che si annullano per $\omega = 3$

POLI : $\pm j\omega_d = \pm j3$ $\qquad C(s) = \dfrac{b_2 s^2 + b_1 s + b_0}{(s^2+9)}$

$P(s) = \dfrac{4}{s+2}$

1) poli dominanti in $-2 \pm j$ → polinomio desiderato ...

2) $K_p = L(0) = C(0)\,P(0) = 4$

3) $e_r = 0$ in condizioni nominali (senza disturbi)

2) $L(0) = \dfrac{4 b_0}{9 \cdot 2} = 4 \qquad b_0 = 18$

1) $$1 + \frac{4(b_2 s^2 + b_1 s + 18)}{(s^2+9)(s+2)} = 0 \Rightarrow p_c(s) = (s^2+9)(s+2) + 4b_2 s^2 + 4b_1 s + 72 =$$

$$= s^3 + (2+4b_2)s^2 + (9+4b_1)s + 90$$

$$p_d(s) = (s+2-j)(s+2+j)(s+c) \underset{c \gg 2}{=} s^3 + (4+c)s^2 + (4c+5)s + 5c \,.$$

perchè di ordine 3

$$\begin{cases} 5c = 90 \rightarrow c = 18 \\ 4c+5 = 9+4b_1 \rightarrow b_1 = 17 \\ 4+c = 2+4b_2 \rightarrow b_2 = 5 \end{cases} \qquad C(s) = \frac{5s^2 + 17s + 18}{s^2+9}$$

3) $K_p = L(0) \rightarrow +\infty \qquad e_r = \dfrac{1}{1+K_p}$ ma $K_p = 4$ non posso usarlo

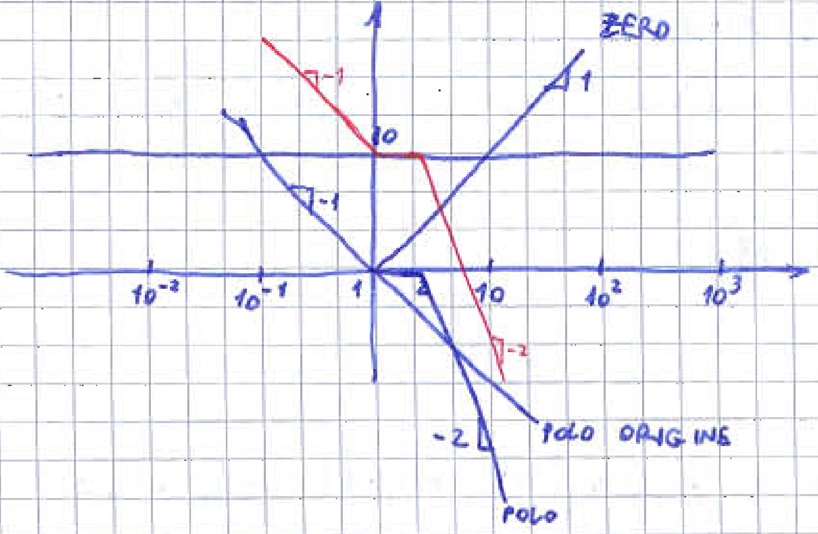
Allora impongo $T_{ry}(0) = 1$ cioè quello che entra è uguale a quello che esce ⇒ errore nullo.

$$T_{ry}(s) = F \cdot \frac{L(s)}{1+L(s)} \qquad T_{ry}(0) = F \cdot \frac{L(0)^{\nearrow 4}}{1+L(0)_{\searrow 4}} = 1 \qquad F \cdot \frac{4}{5} = 1 \Rightarrow F = \frac{5}{4}$$

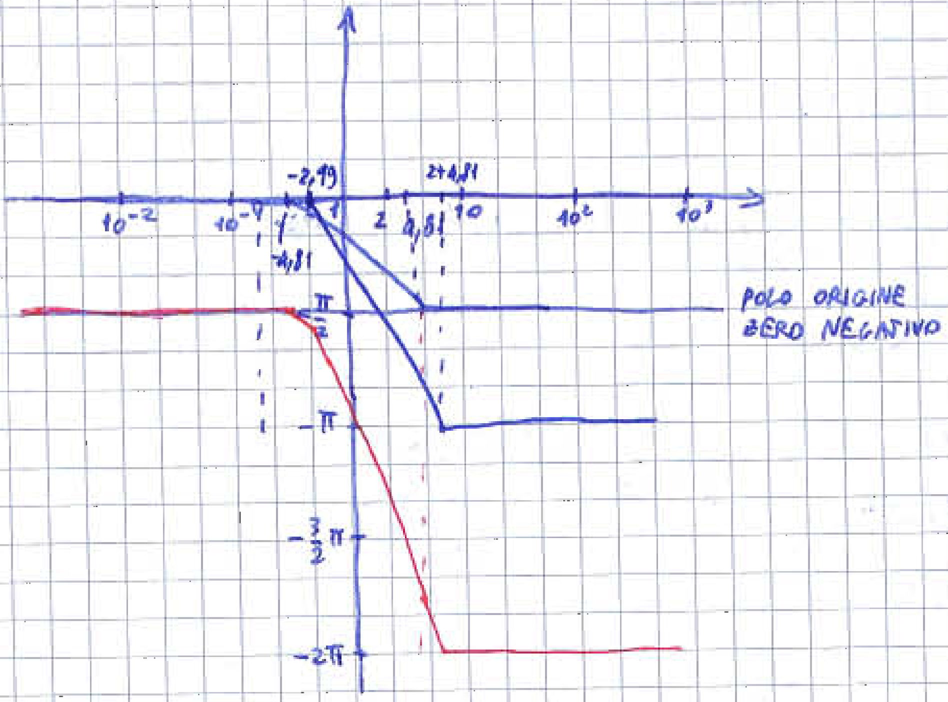
BODE

$$G(s) = \frac{40(1-s)}{s(s+2)^2}$$

$$G(j\omega) = 40 \cdot \frac{1-j\omega}{\underset{\underset{4j\omega\left(1+\frac{j\omega}{2}\right)^2}{\parallel}}{j\omega(j\omega+2)^2}} = 10 \cdot \frac{1-j\omega}{j\omega\left(1+\frac{j\omega}{2}\right)^2}$$



AMPIEZZA



FASE